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Two Studies in Arithmetic

A DIAGNOSTIC STUDY OF EFFICIENCY IN ARITHMETIC

TEACHING THE NUMBER FACTS OF ADDITION

by

HENRY LESTER SMITH

MERRILL THOMAS EATON

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School of Education
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A Diagnostic Study of Efficiency in Arithmetic

By

HENRY LESTER SMITH and MERRILL T. EATON

INTRODUCTION

THE purpose of this study is twofold: first, to make, by means of a mechanical device, an objective and accurate group diagnosis of the efficiency of a group of 77 fourth grade pupils in the basic number facts of addition, subtraction, multiplication, and division; and, second, to make an individual diagnostic study, also by mechanical device, of those pupils who show the least efficiency, according to the group diagnosis.

Group diagnosis offers advantages over individual diagnosis because of the great saving in time, and diagnosis by mechanical device offers objectivity and therefore greater accuracy than diagnosis by pencil and paper tests. Efficiency or lack of efficiency in the basic facts of the fundamental processes of arithmetic may be readily discovered by standardized pencil and paper tests, but such tests have not proved entirely satisfactory in that they fail to measure or portray adequately the mental processes of the pupils. The desirability, accuracy, and objectivity of the mechanical device[1] has been demonstrated in individual diagnosis; however, the mechanical device for group diagnosis has received little consideration from teachers of arithmetic or investigators in the field.

In recent years teachers of arithmetic have given considerable attention to the study of the mental activities involved in acquiring efficiency in the fundamental processes. There seems to be a very definite trend at the present toward devising methods of group diagnosis which not only measure efficiency accurately but also portray the mental processes of the pupils in number activity. Individual diagnosis of pupil ability in arithmetic is highly desirable, but it consumes more time than the busy classroom teacher may devote to this phase of teaching. If, however, group diagnosis by mechanical methods provides accurate information concerning both pupil skill and mental processes, so that only those who need it most may be selected for individual diagnosis, the efficiency of instruction in arithmetic should be greatly enhanced. To learn whether or not a mechanical device may be used as a satisfactory tool for group diagnosis of efficiency in the fundamentals of arithmetic is thus as significant in this investigation as are the results of the diagnosis.

The teaching of the fundamentals of arithmetic is relatively well standardized, yet studies of failures of pupils in the elementary school have shown that more pupils fail in arithmetic[2] than in any other elemen-

[1] Eaton, Merrill T. "The Value of the Dictaphone in Diagnosing Difficulties in Addition." In *Twenty-fifth Annual Conference on Educational Measurements*, Indiana University, Bulletin of the School of Education, Vol. XIV, No. 4, Bloomington, 1938, pp. 5-10.

[2] Judd, Charles H *Psychological Analysis of the Fundamentals of Arithmetic*. University of Chicago Press, 1927. 121 pp.

tary subject. Failures are not limited entirely to the low ability group, as many pupils who show satisfactory progress in their other subjects fail in arithmetic. The wide range of individual differences in arithmetic ability among elementary pupils seems to increase with maturity.[3] The failure of the teacher to understand the mental processes of the pupils in their arithmetic activities must be responsible in part for this situation.

A study by Buswell and John[4] of the types of errors made by 414 elementary school pupils, grades 3 to 6, in addition, subtraction, multiplication, and division reveals that lack of knowledge of the fundamental combinations is the major reason for inefficient performance. Numerous investigations of adult efficiency in the four fundamentals have also shown a woeful lack of knowledge of these combinations.

Brueckner[5] has indicated that inability to execute number combinations successfully results from the failure of the pupil to secure an adequate knowledge of the basic number facts during his elementary school period. Brueckner further emphasizes the fact that faulty habits of executing number combinations may keep a pupil's level of efficiency considerably below his possible maximum level, even though pencil and paper test scores may indicate he is making satisfactory progress. Because a pupil may be rewarded for correct answers, he is likely to be deceived into believing that the use of faulty habits is a satisfactory substitute for complete mastery of basic number combinations.

The number and variety of bad habits which many pupils employ when attempting to do arithmetic work is further evidence of the inadequacy of instruction in the basic number combinations. In the field of addition,[6] for example, "dot adders," "counters," "toe tappers," "lip adders," and "vocal adders" may be found in numbers. Slow, inaccurate, and roundabout methods are equally numerous in subtraction, multiplication, and division. The lack of uniformity in methods of teaching the basic number combinations further complicates instruction in arithmetic. Authors of books on methods of teaching arithmetic fail to agree on the procedures the pupil should employ and master in learning to add, subtract, multiply, and divide efficiently.

The research literature dealing with various aspects of arithmetic is extensive. In 1925 Buswell and Judd[7] published a summary of the educational investigations relating to arithmetic, which includes all the available investigations which the authors considered as research or critical discussions of some phase of arithmetic. Each year since 1925 a supplement to the original bibliography has been published by Buswell in the *Elementary School Journal*. The supplements, as well as the

[3] Smith, Henry Lester, and Eaton, Merrill T. "An Analysis of Efficiency in Addition." In *Proceedings of the Twenty-fourth Annual Conference on Educational Measurements*. Indiana University, Bulletin of the School of Education, Vol. XIII, No. 4, Bloomington, 1937. pp. 52-78.

[4] Buswell, G. T., and John, Lenore. "Diagnostic Studies in Arithmetic." *Supplementary Educational Monograph*, No. 27. University of Chicago, 1926. pp. 136-9.

[5] Brueckner, L. J. *Diagnostic and Remedial Teaching in Arithmetic*. John C. Winston Co., Philadelphia, 1930.

[6] Smith, Henry Lester, and Eaton, Merrill T. "An Analysis of Efficiency in Addition." In *Proceedings of the Twenty-fourth Annual Conference on Educational Measurements*. Indiana University, Bulletin of the School of Education, Vol. XIII, No. 4, Bloomington, 1937. pp. 52-78.

[7] Buswell, G. T., and Judd, C. H. *Summary of Educational Investigations Relating to Arithmetic*. University of Chicago Press, 1925. 212 p.

original publication, contain a critical discussion of many of the investigations in addition to the annotated bibliography.

The *Twenty-ninth Yearbook of the National Society for the Study of Education*[8] consists of the report of the society's Committee on Arithmetic. A bibliography of studies concerning arithmetic is included and several of the studies are analyzed and discussed.

A survey of the research literature shows little uniformity in methods of diagnosis in arithmetic, and the results of the diagnostic studies show considerable variation. The survey of the literature does, however, show that there has been little attempt at group diagnosis through the use of mechanical devices.

In individual diagnosis, Eaton[9] has employed the dictaphone in studying the adding efficiency of a number of elementary school pupils. Smith and Eaton[10] have also employed a mechanical device in the study of the relation of accuracy to speed in addition. Such mechanical devices as have been used in diagnostic studies have proved entirely satisfactory for individual study but consume too much time for the diagnosis of large groups of pupils.

Although there is general agreement that the basic processes underlying efficiency in arithmetic activities are inadequately learned, little if any attempt, except by pencil and paper tests, has been made to remedy this situation by objective and accurate group diagnosis.

The investigation is divided into two parts: (1) group diagnosis, and (2) individual diagnosis. The experimental conditions for the group diagnosis are as follows:

EXPERIMENTAL CONDITIONS FOR GROUP DIAGNOSIS

Apparatus and materials.—The mechanical device selected for group diagnosis is the "number master"[11] shown in Figure 1.

The number master is an electrical machine which automatically exposes number combinations in a fixed, predictable sequence. The machine is equipped with a timing motor and a reostat, so that the time of exposure of the number combinations may be adapted to the age, grade, and ability level of the subjects. Although the time of exposure may be short (3 seconds in this study) the machine is so constructed that there is sufficient time (5 seconds) between exposures to permit subjects to write down the answers.

The over-all dimensions of the number master are 11"x18½"x19¼". The cartridges, the boxes containing the cards on which are printed the number combinations, are 4"x5"x16½". Cartridges are easily inserted or removed, and the cards on which the number combinations are printed are marked so that neither the subjects nor the operator experiences any difficulty in following the sequence of problems. When

[8] Whipple, Guy Montrose, ed. *The Twenty-ninth Yearbook of the National Society for the Study of Education.* Public School Publishing Co., Bloomington, Illinois, 1930. 749 pp.

[9] Eaton, Merrill T. "The Value of the Dictaphone in Diagnosing Difficulties in Addition." In *Twenty-fifth Annual Conference on Educational Measurements,* Indiana University, Bulletin of the School of Education, Vol. XIV, No. 4, Bloomington, 1938. pp. 5-10.

[10] Smith, Henry Lester, and Eaton, Merrill T. "The Relation of Accuracy to Speed in Addition." In *Two Studies in Education,* Indiana University, Bulletin of the School of Education, Vol. XIV, No. 1, Bloomington, 1938 pp. 5-23.

[11] The number master is manufactured by the Educational Devices, Inc., Greencastle, Indiana. Information concerning the machine may be obtained by writing John B. Boyd, 210 Hilldale Avenue, Greencastle, Indiana.

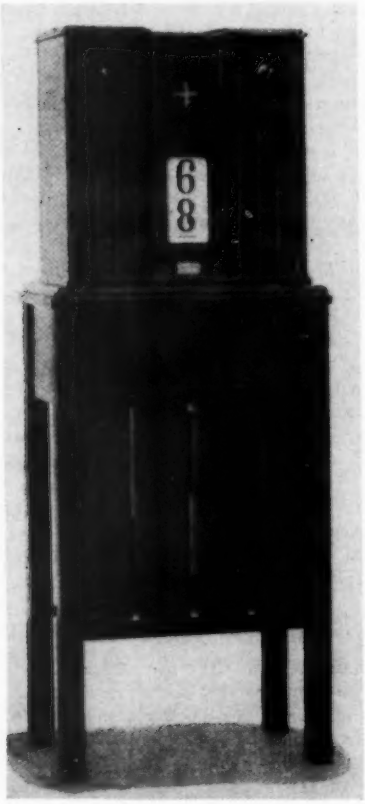


Fig. 1. The Number Master.

the machine is in operation, the cartridge turns end to end. The cards are on both sides, so that with each turn of the cartridge a new combination appears. For each of the four basic operations—addition, subtraction, multiplication, and division—there are two cartridges with 50 exposures (number combinations) in each cartridge. Thus the number combinations in each of these four operations are divided into two tests and designated as Group 1 and Group 2 (G1 and G2). As there are only 90 basic division facts, the five most difficult combinations in each division group were repeated in order to maintain 50 exposures in each cartridge. The number combinations were arranged in each cartridge in what was considered to be the order of difficulty; that is, from the least to the most difficult.

Several different methods have been devised to determine the difficulty of the number combinations. One method, which has been used

extensively, attempts to determine the difficulty of the various combinations on the basis of the number of times the combination is incorrectly solved. Phelps[12] used this method in analyzing the test results of 230 eighth grade children. He found that, in general, there was a regular increase in the number of errors as the combinations increased in size. Certain combinations not included in the larger digits were so frequently missed that they were designated as "type errors." Phelps arranged an order of difficulty for these combinations.

The most extensive investigation to determine the difficulty of combinations by using the frequency of errors as a criterion has been made by Clapp.[13] Clapp's subjects were 7,000 pupils from grades 4 to 8 inclusive. From this investigation he organized the combinations in the order of difficulty for each grade and for the five grades combined. For the five grades combined, Clapp found that $8+5$ was the most difficult and $7+9$ was second in difficulty.

Another method that has been used to determine the relative difficulty of addition combinations is to measure the time required to give the sum of each combination. The argument in favor of this method is that, although counting may be resorted to in order to get a correct response, counting will increase the reaction time and will thereby show that the combination could not be readily given. Thus, the reaction timing method is more analytic than the "number of errors" method of scoring. In examining the results of Heilman and Shultis[14] and Smith,[15] who used the reaction time method, it is interesting to note that all except three of the combinations in the most difficult third found by the reaction method likewise appear in the most difficult third of Clapp's list.

A third method of determining the difficulty of combinations involves a study of the time required to learn. A fourth method involves the measurement of the degree of retention of the combinations after a period of nonuse. Holloway[16] has reported a study by the latter method. The use of these methods involves considerable difficulty in providing adequate control and therefore the results obtained through their use are open to question.

Browne[17] illustrates a fifth method in which the degree of difficulty of the combinations is determined by the introspective reports of the pupils. Browne centers his investigation on the combinations which his subjects reported to be most difficult, rather than on all the combinations.

The number combinations in addition, subtraction, multiplication, and division and the order in which these combinations were presented by the number master are shown in Figure 2.

[12] Phelps, C. L. "A Study of Errors in Tests of Adding Ability." *Elementary School Teacher* 14:29-39, September, 1913.

[13] Clapp, Frank L. *The Number Combinations: Their Relative Difficulty and the Frequency of Their Appearance in Text-Books.* University of Wisconsin, Bureau of Educational Research, Bulletins No. 1 and 2, Madison, 1924. 20 and 126 pp.

[14] Heilman, J. D., and Shultis, Frank W. *A Study in Addition.* Colorado State Teachers College, Research Bulletin No. 1, Greeley, 1916. 16 pp.

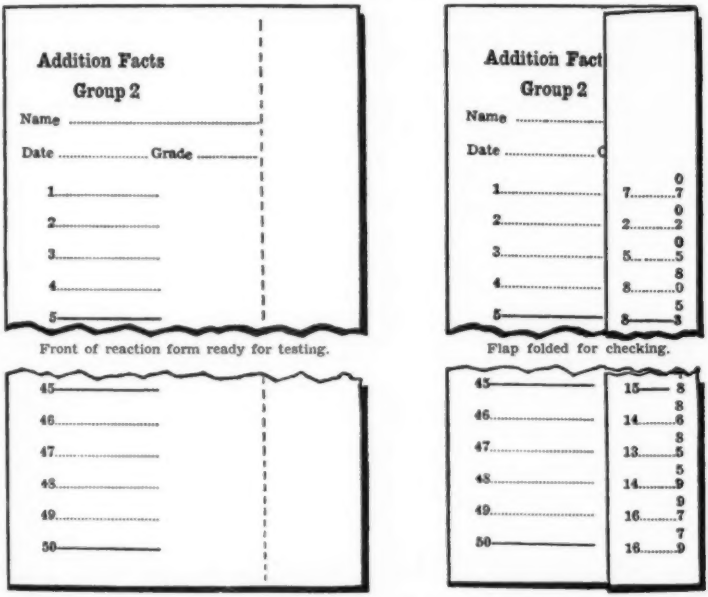
[15] Smith, Nila B. "An Investigation of the Uses of Arithmetic in the Out-of-School Life of First Grade Children." *Elementary School Journal* 24:621-6, April, 1924.

[16] Holloway, Harry Vance. *An Experimental Study to Determine the Relative Difficulty of the Elementary Number Combinations in Addition and Multiplication.* University of Pennsylvania, Doctor's Thesis, Philadelphia, 1914. 102 pp.

[17] Browne, Charles E. "The Psychology of the Simple Arithmetical Processes: A Study of Certain Habits of Attention and Association." *American Journal of Psychology* 18:1-37, January, 1906.

Addition Facts Group 1		Addition Facts Group 2		Subtraction Facts Group 1		Subtraction Facts Group 2		Multiplication Facts Group 1		Multiplication Facts Group 2		Division Facts Group 1		Division Facts Group 2	
1	7	0	4	0	7	2	16	1	2	3	9	4	3	30	56
1 1	26 1	1 7	26 8	1 0	26 1	1 0	26 8	1 6	26 7	1 0	26 5	1 2	26 1	1 6	26 8
0	4	0	6	3	6	10	11	6	9	6	9	25	81	0	63
2 0	27 5	2 0	27 3	2 3	27 1	2 4	27 7	2 1	27 2	2 4	27 9	2 5	27 9	2 2	37 9
2	6	0	4	4	8	10	13	3	2	0	3	12	32	0	36
3 2	28 6	3 5	28 7	3 4	28 4	3 3	28 7	3 1	28 3	3 4	28 9	3 2	28 8	3 3	38 4
5	6	8	9	1	8	11	12	8	3	7	5	10	36	0	1
4 5	29 0	4 0	29 5	4 1	29 8	4 5	29 7	4 1	29 2	4 3	29 8	4 2	29 6	4 9	29 1
2	3	5	7	7	4	9	13	0	4	7	4	10	45	27	54
5—1	30—3	5—3	30—5	5—7	30—3	5—2	30—6	5—0	30—5	5—5	30—8	5—5	30—5	5—9	30—6
6	1	2	9	4	6	9	18	1	4	0	9	9	40	0	6
6 1	31 9	6 9	31 8	6 2	31 3	6 5	31 9	6 5	31 2	6 7	31 3	6 3	31 5	6 4	31 6
1	5	0	3	3	5	7	12	7	3	0	7	6	20	0	2
7 5	32 4	7 8	32 9	7 2	32 2	7 4	32 3	7 1	32 5	7 1	32 7	7 2	32 4	7 5	32 1
1	5	8	9	5	6	9	11	1	2	3	8	8	7	9	28
8 8	33 2	8 3	33 3	8 5	33 4	8 7	33 8	8 1	33 3	8 6	33 9	8 2	33 7	8 9	33 7
8	9	2	6	5	8	7	13	4	5	8	8	12	21	8	56
9 1	34 9	9 6	34 9	9 1	34 6	9 2	34 9	5	3	9	4	9	34 3	9 1	34 7
4	7	6	7	2	9	9	14	5	2	9	9	15	12	7	42
10—4	35—2	10—5	35—6	10—2	35—0	10—4	35—5	10—1	35—4	10—0	35—8	10—5	35—4	10—1	35—7
1	2	5	8	2	10	8	14	1	3	5	4	16	5	40	48
11 3	36 4	11 6	36 9	11 1	36 2	11 2	36 8	11 4	36 4	11 0	36 7	11 4	36 1	11 8	36 6
1	2	7	9	3	7	11	13	5	4	0	4	14	18	72	36
12 7	37 8	12 4	37 4	12 1	37 5	12 2	37 8	12 2	37 3	12 3	37 9	12 2	37 3	12 9	37 9
1	4	8	5	5	12	14	11	1	4	3	6	14	35	2	24
13 4	38 0	13 2	38 7	13 4	38 6	13 7	38 3	13 3	38 3	13 8	38 7	13 7	38 5	13 2	38 3
2	1	0	8	4	6	6	15	1	0	6	7	16	0	18	63
14 0	39 0	14 9	39 7	14 1	39 0	14 5	39 6	14 7	39 5	14 5	39 6	14 2	39 8	14 2	39 7
1	6	2	6	9	6	10	15	2	3	5	7	21	24	24	27
15—6	40—4	15—7	40—7	15—1	40—2	15—6	40—9	15—1	40—7	15—9	40—4	15—7	40—6	15—4	40—3
8	4	0	9	8	3	9	15	1	4	4	6	20	30	18	24
16 8	41 3	16 6	41 6	16 1	41 0	16 3	41 7	16 8	41 6	16 0	41 8	16 5	41 5	16 6	41 8
7	2	4	4	9	10	8	13	2	0	8	9	12	0	4	54
17 7	42 5	17 6	42 9	17 9	42 8	17 3	42 5	17 2	42 9	17 0	42 4	17 3	42 7	17 4	42 9
9	6	4	5	6	5	12	17	1	5	7	8	16	0	45	28
18 1	43 2	18 2	43 8	18 6	43 0	18 9	43 9	18 2	43 4	18 0	43 6	18 8	43 1	18 9	43 4
5	2	3	6	10	8	12	15	9	4	0	7	8	35	8	48
19 1	44 3	19 8	44 8	19 1	44 0	19 8	44 8	19 1	44 5	19 6	44 8	19 4	44 7	19 8	44 8
3	7	3	7	7	1	11	14	2	4	6	8	15	4	3	64
20—1	45—0	20—6	45—8	20—6	45—0	20—6	45—6	20—9	45—4	20—0	45—8	20—3	45—1	20—3	45—8
5	9	3	8	8	7	12	17	7	0	2	8	6	45	32	54
21 0	46 2	21 4	46 6	21 7	46 0	21 4	46 8	21 2	46 2	21 0	46 7	21 1	46 5	21 4	46 6
3	9	7	8	9	4	11	14	8	6	6	6	0	36	42	1
22 0	47 0	22 3	47 5	22 8	47 0	22 9	47 9	22 2	47 3	22 6	47 9	22 6	47 6	22 6	47 1
4	0	3	5	5	10	9	16	2	1	6	7	18	32	5	36
23 1	48 4	23 7	48 9	23 3	48 9	23 6	48 7	23 5	48 1	23 7	48 9	23 9	48 8	23 5	48 4
1	0	3	9	10	7	12	16	6	5	8	9	6	81	49	63
24 2	49 3	24 5	49 7	24 5	49 3	24 5	49 9	24 2	49 6	24 3	49 7	24 3	49 9	24 7	49 9

Procedure.—In order not to interfere with the regular classwork of the children, the principal[18] of Hunter School placed a small but well-lighted room at the disposal of the experimenter. Because of the size of the room it was necessary to test the pupils in groups of 10. The number master was placed on a table of sufficient height so that, as the number combination cards appeared, they could be seen easily by each subject. Careful instructions were given to subjects concerning what they were expected to do and how they were to proceed. The machine was run briefly before each group so that the children understood its operation and knew the amount of time of each exposure and the length of the period they would have for recording their answers. Each subject was then given a printed "reaction form"[19] on which to record his answers. A sample of the reaction form is shown in Figure 3.



Front of reaction form ready for testing.

Flap folded for checking.

Fig. 3. Reaction Form.

The subjects were instructed to leave a blank space after the number of each problem for which they did not know the answer. As shown in Figure 3, the reaction form is organized to facilitate rapid and accurate scoring. When the right-hand edge of the sheet is folded over, the correct answer appears exactly opposite the pupil's answer.

In scoring the reaction forms, omission of an answer was scored as an incorrect answer. Since the subject had been instructed to answer all problems, it was assumed that an omission indicated that he did not

[18] Mr. Frederick Guy Neel, Principal, Hunter School, Bloomington, Indiana.
[19] The reaction forms are printed and distributed by the Educational Devices, Inc., Greencastle, Indiana.

know the correct solution. The order of presentation of problems was addition, subtraction, multiplication, and division. Short rest periods were given at the end of each 50 problems while the experimenter changed cartridges and collected the reaction forms. Pupils were tested on only one series (100 problems) of basic facts on any one day.

Subjects.—The subjects for this study were an unselected group of 77 fourth grade pupils. The subject group consisted of all the pupils enrolled in grades 4A and 4B at the Hunter School, Bloomington, Indiana, including 40 girls and 37 boys ranging in age from 9 to 11 years.

RESULTS OF GROUP DIAGNOSIS

Individual scores.—Table I gives the number of combinations in the two groups of 50 combinations in addition, in subtraction, in multiplication, and in division that were solved correctly by each of the 77 subjects. In the first column, headed G1 under each fundamental process, are the scores for the first 50 combinations.

TABLE I. SCORES MADE BY 77 FOURTH GRADE PUPILS ON THE 100 BASIC FACTS OF ADDITION SUBTRACTION, MULTIPLICATION, AND DIVISION

Subject	Addition			Subtraction			Multiplication			Division		
	G1	G2	T	G1	G2	T	G1	G2	T	G1	G2	T
1	50	50	100	49	42	91	49	47	96	40	46	86
2	42	35	77	31	6	37	46	32	78	36	28	64
3	49	47	96	50	47	97	46	37	83	50	47	97
4	39	47	86	50	17	67	48	34	82	33	28	61
5	50	50	100	50	48	98	50	47	97	50	46	96
6	50	50	100	50	48	98	47	27	74	36	38	74
7	47	32	79	45	8	53	29	1	30	12	10	22
8	50	49	99	50	47	97	50	50	100	45	39	84
9	50	45	95	49	27	76	37	12	49	19	24	43
10	49	50	99	47	38	85	50	43	93	45	44	89
11	50	49	99	50	46	96	46	41	87	35	46	81
12	50	39	89	47	24	71	45	32	77	44	47	91
13	50	47	97	50	50	100	50	46	96	45	41	86
14	50	42	92	50	37	87	49	42	91	37	41	78
15	42	38	80	42	41	83	44	28	72	50	41	91
16	50	47	97	48	22	70	46	15	61	31	38	69
17	42	9	51	4	4	8	28	2	30	1	3	4
18	50	44	94	47	28	75	49	39	88	41	22	63
19	48	48	96	41	45	86	49	42	91	45	41	86
20	49	50	99	50	50	100	50	48	98	41	45	86
21	50	43	93	50	42	92	50	42	92	49	33	82
22	50	47	97	50	38	88	48	38	86	41	34	75
23	50	47	97	50	50	100	50	49	99	50	50	100
24	50	49	99	50	48	98	47	45	92	49	50	99
25	49	45	94	50	32	82	49	43	92	50	46	96
26	50	39	89	50	37	87	49	8	57	48	34	82
27	46	48	94	50	30	80	44	18	62	35	11	46
28	42	49	91	42	49	91	50	47	97	50	48	98
29	49	45	94	50	48	98	49	49	98	50	48	98
30	49	49	98	44	47	91	49	49	98	40	43	83
31	50	47	97	50	46	96	50	46	96	48	48	96
32	49	42	91	49	5	54	43	18	61	40	9	49
33	50	45	95	42	26	68	50	49	99	44	43	87
34	49	47	96	43	47	90	48	36	84	47	35	82
35	36	33	69	36	5	41	18	0	18	8	3	11
36	47	47	94	39	43	82	50	45	95	47	37	84
37	50	47	97	50	49	99	46	36	82	47	50	97
38	41	45	86	41	47	88	47	23	70	38	30	68
39	50	45	95	50	49	99	50	49	99	46	41	87
40	50	43	93	50	44	94	43	30	73	40	25	65

TABLE I. (Continued)

SUBJECT	Addition			Subtraction			Multiplication			Division		
	G1	G2	T	G1	G2	T	G1	G2	T	G1	G2	T
41	48	42	90	31	26	57	44	21	65	21	0	21
42	50	45	95	50	41	91	47	34	81	43	50	93
43	49	26	75	48	18	66	33	20	53	20	13	33
44	49	49	98	50	50	100	49	46	95	47	48	95
45	50	44	94	50	50	100	50	12	62	48	49	97
46	50	37	87	39	18	57	43	8	51	31	17	48
47	50	46	96	50	50	100	50	49	99	50	47	97
48	48	45	93	50	33	83	42	3	45	18	12	30
49	47	33	80	44	29	73	38	19	57	32	14	46
50	50	46	96	50	42	92	49	26	75	45	33	78
51	15	17	32	45	44	99	47	47	94	26	42	68
52	42	46	88	32	16	48	27	15	42	17	29	46
53	49	50	99	50	21	71	49	12	61	22	23	45
54	50	47	97	50	38	88	50	49	99	40	39	79
55	49	43	92	48	12	60	46	24	70	39	30	69
56	50	48	98	50	26	76	45	34	79	34	32	66
57	50	46	96	50	42	92	49	43	92	49	42	91
58	50	49	99	50	47	97	49	49	98	50	49	99
59	49	46	95	49	45	94	46	31	77	43	37	80
60	50	50	100	50	43	93	50	49	99	49	44	93
61	50	48	98	40	25	65	50	48	98	21	12	33
62	50	50	100	50	50	100	49	49	98	50	50	100
63	50	47	97	50	42	92	50	38	88	36	30	66
64	50	47	97	50	37	87	50	49	99	46	47	93
65	50	45	95	36	34	70	46	49	95	39	30	69
66	47	44	91	50	24	74	43	33	76	10	21	31
67	50	48	98	50	50	100	48	49	97	50	50	100
68	49	49	98	42	45	87	46	49	95	45	38	83
69	50	45	95	50	29	79	44	29	73	27	26	53
70	50	48	98	42	48	90	50	49	99	49	49	98
71	50	48	98	50	43	93	48	48	96	49	42	91
72	50	49	99	50	48	98	46	36	82	47	40	87
73	50	46	96	50	41	91	46	34	80	45	41	86
74	50	48	98	44	31	75	50	34	84	34	32	66
75	50	47	97	50	44	94	44	24	68	32	39	71
76	38	50	88	50	44	94	46	25	71	41	29	70
77	44	48	92	50	46	96	46	34	80	50	46	96
Average	48	44	92	46	37	83	47	34	81	39	35	74

The range in scores in addition is from 15 to 50 for the first group and from 9 to 50 for the second group. For the first 50 combinations the highest score was approximately 3½ times as high as the lowest, while in the second group the highest score was approximately 5½

TABLE II. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGE SCORES MADE ON THE FIRST AND SECOND LISTS OF 50 COMBINATIONS

	Addition		Subtraction		Multiplication		Division	
	G1	G2	G1	G2	G1	G2	G1	G2
S. D.	4.50	6.15	7.25	11.65	5.45	13.70	10.96	13.20
S. D. Average	.51	.70	.83	1.33	.62	1.56	1.25	1.51
Average	48.00	44.00	46.00	37.00	47.00	34.00	39.00	35.00
Difference	4		9		13		4	
S. D. Difference	.87		1.57		1.68		1.96	
Difference / S. D. Difference	4.60		5.73		7.74		2.55	

times as great as the lowest. The range of scores in the second group of problems indicates that the subjects did not know the combinations of G2 as well as they did those of G1. This contention is substantiated when the average scores are considered, for the average scores for G1 and G2 in addition were 48 and 44, respectively, a difference of 4. This difference is a statistically reliable difference (Table II).

The range of scores for both the first and second series of problems in subtraction was 4 to 50, the highest score being 12½ times as great as the lowest. This is a much wider range of scores than was found in the addition problems. The fact that the range of scores was the same for the two groups of subtraction combinations does not indicate, however, that they were of equal difficulty to the subjects, for the average score for G1 was 46 and for G2 it was 37. This difference of 9 between the two averages is a statistically reliable difference (Table II).

The range of scores for the first 50 problems in multiplication was 18 to 50, and for the second series it was 0 to 50. The average score for G1 was 47 and for G2 it was 34, a difference of 13. The difference between the two averages is statistically reliable (Table II). This fact shows that in multiplication, as in subtraction and addition, the problems of G2 were not as well known as were the combinations of G1.

The range of scores in division was larger than that in the other fundamental processes; for G1 the range was 1 to 50 and for G2, 0 to 50. The difference between the averages (G1, 39, and G2, 35) for the two series was 4, which, although possessing high reliability (Table II), is not completely reliable. The reliability, however, is sufficiently high to indicate that the subjects found the second series of problems in division more difficult than the first.

In each of the four types of problems there was greater variability of scores in the second series of problems than in the first series, as is shown by comparing the standard deviation (Table II). In each case the S.D.'s were higher for the second group of problems than for the first.

Further evidence of the greater difficulty of the second series of problems may be obtained by examining the distribution of scores in Table III.

TABLE III. DISTRIBUTION OF SCORES OF THE 77 SUBJECTS IN EACH OF THE FOUR FUNDAMENTALS

Scores	Addition		Subtraction		Multiplication		Division	
	G1	G2	G1	G2	G1	G2	G1	G2
41-50	73	66	68	45	70	35	43	36
31-40	3	8	8	10	3	17	21	16
21-30	0	1	0	12	3	10	5	14
11-20	1	1	0	5	1	9	5	6
0-10	0	1	1	5	0	6	3	5

On G1 in addition, 73 of the 77 subjects made a score of 41-50, while only 66 subjects ranked as high on G2. In subtraction there were 68 subjects who made a score of 41 or better on G1, but on G2 the num-

ber making similar scores had dropped to 45. In multiplication there were exactly twice as many subjects making scores from 41 to 50 on G1 (70) as there were on G2 (35). On G1 in division, 43 pupils made scores of 41 to 50 while on G2 only 36 made such scores.

From the comparison of the S.D.'s, the range of scores, the distribution of scores, and the differences between the averages of the first and second groups of problems, it is clearly shown that for the subjects in this investigation the combinations of the G2 problems have not been mastered to the same degree of efficiency as have those problems of the G1 series.

Because of the relatively narrow range of scores in the G1 basic facts of addition, subtraction, multiplication, and division, correlations were not computed. However, an examination of the individual scores shows a high constancy of performance between the G1 and G2 series of problems; that is, in most cases the pupil scoring high on the G1 problems also did well on the G2 problems although his score on these was usually somewhat lower than on the G1 problems.

Total scores.—The total scores made by each subject on G1 and G2 for each of the four types of problems are given in Table I. The range of total scores for addition was from 32 to 100; for subtraction, from 8 to 100; for multiplication, from 18 to 100; and for division, from 4 to 100. The greatest range of scores is found in division and subtraction, with a smaller range in multiplication and a relatively narrow range in addition.

A comparison of the distribution of the total scores given in Table IV is interesting. In addition, 61 of the 77 pupils made a score of 91 to 100 and only one made less than 41. In subtraction, 37 pupils made a score of more than 91, which was only slightly more than half as many as scored 91 or above in addition. The distribution of scores in multiplication was similar to that of subtraction. Of the 77 pupils, 34 made scores above 91, while 10 pupils made scores below 60. The widest distribution of scores was found in division, for only 24 of the 77 pupils made scores above 91, while 15 pupils made scores of 50 or less.

TABLE IV. DISTRIBUTION OF TOTAL SCORES OF THE 77 SUBJECTS

RANGE	Addition	Subtraction	Multiplication	Division
91-100	61	37	34	24
81-90	8	15	11	17
71-80	5	10	13	7
61-70	1	6	9	13
51-60	1	5	4	1
41-50	0	2	3	7
31-40	1	1	2	3
21-30	0	0	0	3
11-20	0	0	1	1
0-10	0	1	0	1

A study of the distribution of scores indicates that the basic facts of addition were better known than were those of subtraction, multiplication, and division. This contention is substantiated when the relia-

bilities of the differences between averages of the total scores are compared. These data are given in Table V. The total average score for addition was 92 and for subtraction 83. This difference of 9 is a statistically reliable difference. It is interesting to note that the total average scores for subtraction (83) and multiplication (81) were approximately the same. As is shown in Table V, the difference of only 2 between the two averages has a reliability of only slightly greater than chance. There is a rather wide difference (7) between the average scores of multiplication and division. The difference, while not entirely statistically reliable, is sufficiently high to indicate that the basic facts of division were not known as well as those of multiplication.

TABLE V. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGES OF TOTAL SCORES

	Addition	Subtraction	Multiplication	Division
S. D.	6.10	16.15	18.68	22.00
S. D. Average	.70	1.84	2.13	2.51
Average	92.00	83.00	81.00	74.00
Difference		9.00	2.00	7.00
S. D. Difference		1.97	2.82	3.29
Difference / S. D. Difference		4.62	.71	2.16

	Multiplication	Addition	Division	Subtraction
S. D.	18.68	6.10	22.00	16.15
S. D. Average	2.13	.70	2.51	1.84
Average	81.00	92.00	74.00	83.00
Difference		11.00	18.00	9.00
S. D. Difference		2.24	2.61	3.11
Difference / S. D. Difference		4.91	6.90	2.89

There is a statistically reliable difference between the average scores of addition and multiplication (11) and also between the average scores of addition and division (18). The difference of 9 between the average scores for subtraction and division possesses high statistical reliability, although it is not entirely reliable. In examining the variability of the scores, the S.D. for addition is 6.10, while for subtraction it is 16.15. The S.D. for multiplication, 18.68, is not greatly different from that for subtraction, but the S.D. of 22 for division indicates an increase in the variability of the scores.

The superiority of the pupil's performance in addition as compared with subtraction, multiplication, and division may be seen by examining Figure 4. There is a gradual decline in efficiency from addition through division, with the major drops coming between addition and subtraction, and multiplication and division, with only a slight difference between subtraction and multiplication.

The total scores in addition were arranged in order from highest to lowest, and then these 77 scores were divided into four groups. There were 19 subjects in each of the first three quartiles and 20 in the fourth. Addition scores were selected as a basis for the group division because the group as a whole was more efficient in the basic addition facts than in the other fundamental processes. The scores in addition, subtraction,

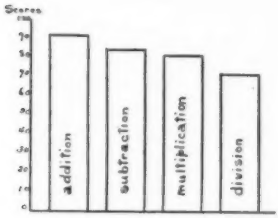


Fig. 4. Total Average Scores Made in Addition, Subtraction, Multiplication, and Division.

TABLE VI. DISTRIBUTION OF SCORES IN QUARTILES BASED ON EFFICIENCY IN ADDITION

SUBJECT	Addition	Subtraction	Multiplication	Division	Average
FIRST QUARTILE					
1	100	91	96	86	93.25
5	100	98	97	96	97.75
6	100	98	74	74	86.50
60	100	93	99	93	96.25
62	100	100	98	100	99.50
8	99	97	100	84	95.00
10	99	85	93	89	91.50
11	99	96	87	81	90.75
20	99	100	98	86	95.75
24	99	98	92	99	97.00
53	99	71	61	45	69.00
58	99	97	98	99	98.25
72	99	98	82	87	91.50
30	98	91	98	83	92.50
44	98	100	95	95	97.00
56	98	76	79	66	79.75
61	98	65	98	33	73.50
67	98	100	97	100	98.75
68	98	87	95	83	90.75
Average	98.95	91.63	91.42	83.11	91.17
SECOND QUARTILE					
70	98	93	99	98	96.25
71	98	93	96	91	94.50
74	98	75	84	66	80.75
13	97	100	96	86	94.75
16	97	70	61	69	74.25
22	97	88	86	75	86.50
23	97	100	99	100	99.00
31	97	96	96	96	96.25
37	97	99	82	97	93.75
54	97	88	99	79	90.75
63	97	92	88	66	85.75
64	97	87	99	93	94.00
75	97	94	68	71	82.50
3	96	97	83	97	93.25
19	96	86	91	86	89.75
34	96	93	84	82	88.00
47	96	100	99	97	98.00
59	96	92	75	78	85.25
57	96	92	92	91	92.75
Average	96.84	91.00	86.26	85.16	89.89

TABLE VI. (Continued)

Subject	Addition	Subtraction	Multiplication	Division	Average
Third Quartile					
73	96	91	80	86	88.25
9	95	76	49	43	65.75
33	95	68	99	87	87.25
39	95	99	99	87	95.00
42	95	91	81	93	90.00
59	95	94	77	80	86.50
65	95	70	95	69	82.25
60	95	79	73	53	75.00
18	94	75	88	63	80.00
25	94	82	92	96	91.00
27	94	80	62	46	70.50
29	94	98	98	98	97.00
36	94	82	95	84	88.75
45	94	100	62	97	88.25
21	93	92	92	82	89.75
40	93	94	73	65	81.25
48	93	83	45	30	62.75
14	92	87	91	78	87.00
55	92	60	70	69	72.75
Average	94.11	84.26	80.00	74.00	83.11
Fourth Quartile					
77	92	96	80	96	91.00
28	91	91	97	98	94.25
32	91	54	61	49	63.75
66	91	74	76	31	68.00
41	90	57	65	21	58.25
12	89	71	77	91	82.00
26	89	87	57	82	78.75
52	88	48	42	46	56.00
76	88	94	71	70	80.75
46	87	57	51	48	60.75
4	86	67	82	61	74.00
38	86	88	70	68	78.00
15	80	83	72	91	81.50
49	80	73	57	46	64.00
7	79	53	30	22	46.00
2	77	37	78	64	64.00
43	75	66	53	33	56.75
35	69	41	18	11	34.75
17	51	8	30	4	23.25
51	32	99	94	68	73.25
Average	80.55	67.20	63.05	55.00	66.15

multiplication, and division for each of the four groups are given in Table VI. These data have been plotted in Figure 5. The trend of the data may be readily seen by examining the bars for each quartile in this figure.

For the first quartile the scores in addition exceeded those in subtraction, multiplication, and division. There was little difference in group efficiency in subtraction and multiplication, but in division there was a considerable drop in efficiency. The trend of these data is quite similar to the trend of the data for the whole group. In the second quartile, the drop in efficiency between addition and subtraction and between multiplication and division is less than it was for the first quartile, but the difference in scores between subtraction and multiplication is greater. The difference in scores from addition to division shows

a uniform and constant drop. In the third quartile, the difference in scores in the basic number facts of the fundamental processes is relatively uniform, but the loss in efficiency from addition through the other three fundamental processes is greater than that for the pupils in the second quartile.

In general the differences in the fourth quartile between the scores for addition, subtraction, multiplication, and division are greater than those found in either the first, second, or third quartiles. In the second, third, and fourth quartiles, the differences in scores between addition and subtraction are greater than between subtraction and multiplication or between multiplication and division. In the first quartile, however, the largest drop in efficiency is between multiplication and division. These data indicate that, when scores are compared on the basis of efficiency in the basic number facts of addition, efficiency in addition will be superior to that of the other fundamental processes. Figure 5 also shows that the less efficient the subjects are in addition, the less able they are to execute the basic number facts in the other three processes.

The total average scores for each quartile are given in Table VI. These total average scores represent the efficiency of the group in the basic number facts of the four fundamental processes. A comparison of these scores shows that the efficiency of the first quartile (91.17) was only 1.28 greater than that of the second quartile (88.89), a difference which has little statistical reliability (Table VII). However, the difference of 6.78 between the total average scores of the second and third quartiles possesses high statistical reliability, and the difference of 16.96 between the third and fourth quartiles is a statistically reliable difference.

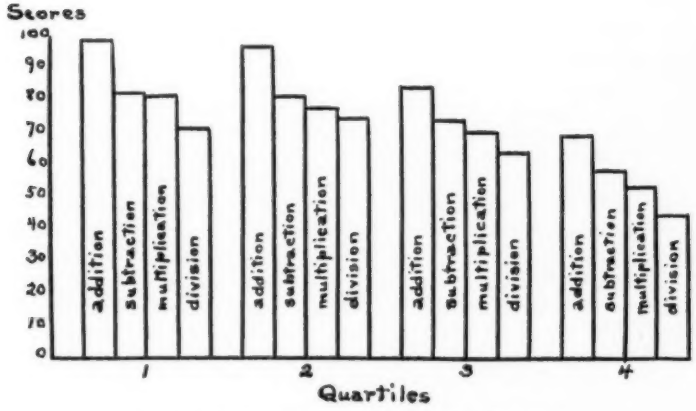


Fig. 5. Distribution of Scores in the Four Quartiles.

TABLE VII. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGE SCORES FOR EACH QUARTILE

	First Quartile	Second Quartile	Third Quartile	Fourth Quartile
S. D.	9.35	7.35	9.95	17.35
S. D. Average	2.14	1.68	2.28	3.88
Average	91.17	89.89	83.11	66.15
Difference		1.28	6.78	16.96
S. D. Difference		2.72	2.83	4.50
Difference / S. D. Difference		.47	2.39	3.77

	Third Quartile	First Quartile	Fourth Quartile	Second Quartile
S. D.	9.95	9.35	17.35	7.35
S. D. Average	2.28	2.14	3.88	1.68
Average	83.11	91.17	66.15	89.89
Difference		8.06	25.02	23.74
S. D. Difference		3.13	4.43	4.23
Difference / S. D. Difference		2.57	5.64	5.61

The variability of the scores as measured by the S.D. is approximately the same for the first and third quartiles (Table VII) when the four fundamental processes are considered. The least variability is found in the second quartile and the greatest in the fourth. The first two quartiles are approximately equal in efficiency. The efficiency of the third quartile is less than that of the first and second, and that of the fourth quartile is definitely inferior to that of the other three. From these results it is again apparent that, as a group, those students who are efficient in addition are also efficient in subtraction, multiplication, and division, but to a lesser extent.

Difficulty of combinations.—The basic number facts of the fundamental processes were presented in the order of difficulty determined by previous studies. A study of the number of times each combination was incorrectly solved by the 77 fourth grade pupils shows considerable deviation from the order of presentation. The combinations are arranged from those which were easiest, according to the results of this study, to those which were most difficult. In Table VIII the number at the left of the combination represents the order in which the 50 addition problems in G1 were presented and the number at the right shows how often each combination was missed by the 77 subjects.

The range of the frequency of the incorrect solutions for the first series of 50 addition combinations is from 0 to 9. The combination 0+0, which was the second problem presented to the group, was the only combination which all of the 77 pupils solved correctly. Therefore it ranks first in order of difficulty. The most difficult combinations were 39 (1+0) and 50 (0+1), each of which was missed 9 times. Combination 21 (5+0) is an example of a combination that was missed less

frequently than its position in the order of presentation indicated, and combination 10 (4+4) is one that was incorrectly solved more frequently than supposedly harder combinations.

TABLE VIII. DIFFICULTY OF THE ADDITION COMBINATIONS OF GROUP 1 AS DETERMINED BY THE NUMBER OF TIMES THE COMBINATIONS WERE INCORRECTLY SOLVED

Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed
2	0+0	0	13	1+4	3	34	9+9	5
1	1+1	1	17	7+7	3	42	2+5	5
21	5+0	1	20	3+1	3	43	6+2	5
3	2+2	2	23	4+1	3	44	2+3	5
5	2+1	2	24	1+2	3	10	4+4	6
14	2+0	2	28	6+6	3	27	4+5	6
16	8+8	2	36	2+4	3	45	7+0	6
19	5+1	2	41	4+3	3	47	9+0	6
22	3+0	2	6	6+1	4	30	3+3	7
25	3+2	2	7	1+5	4	38	4+0	7
26	7+1	2	15	1+6	4	40	6+4	7
29	6+0	2	18	9+1	4	46	9+2	7
31	1+9	2	32	5+4	4	48	0+4	8
35	7+2	2	37	2+8	4	49	0+3	8
4	5+5	3	8	1+8	5	39	1+0	9
11	1+3	3	9	8+1	5	50	0+1	9
12	1+7	3	33	5+2	5			

TABLE IX. DIFFICULTY OF THE ADDITION COMBINATIONS OF GROUP 2 AS DETERMINED BY THE NUMBER OF TIMES THE COMBINATIONS WERE INCORRECTLY SOLVED

Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed
4	8 + 0	3	21	3 + 4	5	11	5 + 6	10
5	5 + 3	3	33	9 + 3	5	34	6 + 9	10
6	2 + 9	3	9	2 + 6	6	38	5 + 7	10
26	4 + 8	3	14	0 + 9	6	37	9 + 4	11
27	6 + 3	3	17	4 + 6	6	42	4 + 9	11
1	0 + 7	4	18	4 + 2	6	43	5 + 8	11
2	0 + 2	4	28	4 + 7	6	30	7 + 5	12
3	0 + 5	4	22	7 + 3	7	44	6 + 8	12
15	2 + 7	4	23	3 + 7	7	48	5 + 9	13
16	0 + 6	4	24	3 + 5	7	40	6 + 7	14
19	3 + 8	4	25	8 + 4	7	41	9 + 6	14
31	9 + 8	4	29	9 + 5	7	45	7 + 8	16
32	3 + 9	4	39	8 + 7	7	46	8 + 6	17
7	0 + 8	5	12	7 + 4	8	47	8 + 5	18
8	8 + 3	5	36	8 + 9	8	49	9 + 7	21
13	8 + 2	5	35	7 + 6	9	50	7 + 9	27
20	3 + 6	5	10	6 + 5	10			

The problems containing zero added to a digit were those missed most frequently in the G1 addition basic facts. The larger digit combinations were missed most frequently in the G2 problems. A study

of the times that certain of the addition combinations were missed presents some interesting data. For example, problem 29 (9+5) was missed 7 times, while combination 48 (5+9) was missed 13 times (Table IX). It seems unreasonable to conclude that 9+5 is an easier

TABLE X. DIFFICULTY OF THE SUBTRACTION COMBINATIONS OF GROUP I AS DETERMINED BY THE NUMBER OF TIMES THE COMBINATIONS WERE INCORRECTLY SOLVED

Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed
1	0 0	0	30	4 3	3	29	8 8	8
11	2 1	1	42	10 8	3	34	8 6	8
12	3 1	1	9	5 1	4	40	6 2	8
14	4 1	1	26	7 1	4	48	10 9	8
23	5 3	1	27	6 1	4	49	7 3	8
24	10 5	1	32	5 2	4	5	7 7	9
7	3 2	2	31	6 3	5	36	10 2	9
16	8 1	2	4	1 1	6	41	3 0	14
20	7 6	2	33	6 4	6	44	8 0	14
21	8 7	2	37	7 5	6	35	9 0	15
22	9 8	2	38	12 6	6	43	5 0	15
25	6 5	2	10	2 2	7	45	1 0	15
28	8 4	2	2	3 3	8	46	7 0	15
6	4 2	3	3	4 4	8	50	10 7	15
13	5 4	3	8	5 5	8	47	4 0	16
15	9 1	3	17	9 9	8	39	6 0	17
19	10 1	3	18	6 6	8			

TABLE XI. DIFFICULTY OF THE SUBTRACTION COMBINATIONS OF GROUP 2 AS DETERMINED BY THE NUMBER OF TIMES THE COMBINATIONS WERE INCORRECTLY SOLVED

Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed
2	10 − 4	8	20	11 − 6	16	35	14 − 5	24
6	9 − 5	8	28	13 − 7	16	46	17 − 8	24
1	2 − 0	9	31	18 − 9	16	33	11 − 8	27
7	7 − 4	9	22	11 − 9	17	44	15 − 8	27
3	10 − 3	11	23	9 − 6	17	45	14 − 6	27
4	11 − 5	11	30	13 − 6	17	34	13 − 9	28
5	9 − 2	11	16	9 − 3	18	39	15 − 9	29
8	9 − 7	11	24	12 − 5	18	43	17 − 9	29
10	9 − 4	11	29	12 − 7	19	47	14 − 9	29
26	16 − 8	11	32	12 − 3	19	37	13 − 8	30
11	8 − 2	12	25	11 − 4	20	42	13 − 5	31
9	7 − 2	13	18	12 − 9	21	48	16 − 7	31
14	8 − 5	13	19	12 − 8	21	41	15 − 7	32
13	14 − 7	14	17	8 − 3	22	40	15 − 9	34
27	11 − 7	14	36	14 − 8	22	49	16 − 9	37
12	11 − 2	15	38	11 − 3	22	50	13 − 4	39
15	10 − 6	15	21	12 − 4	23			

combination than 5+9. Since similar results were found with other combinations, it is assumed that the pupils had had more experience with certain combinations when listed one way than when listed the other.

The number of times each combination in the second series of addition problems was incorrectly solved is given in Table IX. The range of frequencies of the incorrect solutions is from 3 to 27. In G2, as in G1, some of the combinations were out of their order of difficulty for the

TABLE XII. DIFFICULTY OF THE MULTIPLICATION COMBINATIONS OF GROUP 1 AS DETERMINED BY THE NUMBER OF TIMES THE COMBINATIONS WERE INCORRECTLY SOLVED

Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed
1	1 × 6	0	7	7 × 1	2	38	4 × 3	6
5	0 × 0	0	9	4 × 1	2	43	5 × 4	6
6	1 × 5	0	20	2 × 9	2	36	3 × 4	7
8	1 × 9	0	25	2 × 6	2	30	4 × 5	8
11	1 × 5	0	26	2 × 7	2	44	5 × 5	8
14	1 × 7	0	27	9 × 2	2	37	3 × 3	10
16	1 × 8	0	12	5 × 2	3	45	4 × 4	10
18	1 × 2	0	22	8 × 2	3	49	5 × 6	10
10	5 × 1	1	24	6 × 2	3	40	3 × 7	11
13	1 × 3	1	28	2 × 3	3	47	6 × 3	14
15	2 × 1	1	29	3 × 2	3	41	4 × 6	18
17	2 × 2	1	21	7 × 2	4	48	1 × 1	20
19	9 × 1	1	31	4 × 2	4	39	0 × 5	23
23	2 × 5	1	32	3 × 5	4	46	0 × 2	26
2	6 × 1	2	35	2 × 4	4	50	0 × 8	30
3	3 × 1	2	33	2 × 8	5	42	0 × 9	31
4	8 × 1	2	34	5 × 3	6			

pupils studied in this investigation. In relation to the number of times they were missed, approximately half of the combinations in G2 should have been presented in G1. Combinations 26 (4+8) and 27 (6+3) are examples of problems that were less difficult and combinations 10 (6+5) and 11 (5+6) were more difficult than their positions indicated.

The number of times each combination of the G1 subtraction combinations was missed is given in Table X. The range of frequencies of incorrect solutions is from 0 to 17. Problem 1 (0 – 0) was solved correctly by all 77 subjects and, in relation to the number of times missed, was the easiest combination of all. Problem 39 (6 – 0) was missed 17 times and was the most difficult combination for the group. Problem 23 (5 – 3) is an example of a combination that was easier and problem 5 (7 – 7) was more difficult than its position in order of presentation indicated.

The number of times each combination of the G2 subtraction combinations was missed is given in Table XI. The range of frequencies for the incorrect solutions is 8 to 39. Problems 2 (10 – 4) and 6 (9 – 5) were each missed 8 times and were the easiest combinations in the G2 series. Problem 50 (13 – 4) was incorrectly solved 39 times and was therefore the most difficult combination of the 50. In general, the order of difficulty of the G2 combinations as found in this study compares favorably with the order of presentation. A study of the number of times each combination in the two series was incorrectly solved shows that to arrange the combinations in order of difficulty some of the problems in G2 should be moved up to G1, and a like number in G1 should be transferred to G2.

In the G1 subtraction basic facts the frequency of incorrect solutions was high for all combinations involving zero. The difficulty of zero in subtraction combinations is especially demonstrated in combinations in which zero is subtracted from a digit. With the pupils of this investigation, the typical answer to such subtraction problems was to take 1 rather than 0 from the digit. The subtraction of a digit from 10 also proved difficult, which indicates that the subjects were not sufficiently trained in subtraction where zero is involved. The high frequency of the incorrect solutions in the G2 combinations indicates that the pupils have not had sufficient experience with the subtraction combinations containing the larger digits. Either additional drill is needed in these combinations or more efficient methods of teaching must be devised.

The number of times each combination of the G1 multiplication combinations was missed is given in Table XII. The range of frequencies of incorrect solutions is 0 to 31. Problems 1, 5, 6, 8, 11, 14, 16, and 18 were solved correctly by the entire group and were, therefore, of equal difficulty. Problem 42 (9×0), most difficult combination of the 50, was missed 31 times. The order of difficulty of the G1 multiplication combinations as determined by the number of times each combination was missed is not greatly different from the order of presentation.

The number of times each combination of the G2 basic multiplication facts was missed is given in Table XIII. The range of frequencies for incorrect solutions is 10 to 34. Combinations 4 (3×7) and 27 (9×9)

were each missed 10 times and were, therefore, the easiest combinations in G2. Problems 18 (0×7) and 17 (0×8) were the most difficult, each combination being incorrectly solved 34 times. A comparison of the order of presentation with the order of difficulty, as measured by the

TABLE XIII. DIFFICULTY OF THE MULTIPLICATION COMBINATIONS OF GROUP 2 AS DETERMINED BY THE NUMBER OF TIMES THE COMBINATIONS WERE INCORRECTLY SOLVED

Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed
4	7 × 3	10	35	9 × 8	18	6	0 × 7	27
27	9 × 9	10	33	8 × 9	19	19	0 × 6	27
8	3 × 6	12	34	8 × 4	20	49	9 × 7	27
14	6 × 5	12	37	4 × 9	21	12	0 × 3	28
28	3 × 9	12	40	7 × 4	22	48	7 × 9	28
26	9 × 5	13	32	7 × 7	23	1	3 × 0	29
2	6 × 4	14	36	4 × 7	23	47	6 × 9	29
9	8 × 5	14	38	6 × 7	23	50	9 × 6	29
31	9 × 3	14	39	7 × 6	23	11	5 × 0	30
13	3 × 8	16	41	6 × 8	23	25	1 × 0	30
30	4 × 8	16	43	8 × 6	23	10	9 × 0	31
24	8 × 3	16	42	9 × 4	24	21	2 × 0	31
5	7 × 5	17	44	7 × 8	24	16	4 × 0	33
15	5 × 9	18	45	8 × 8	24	20	6 × 0	33
22	6 × 6	18	46	8 × 7	25	17	8 × 0	34
23	3 × 7	18	7	0 × 1	26	18	7 × 0	34
29	5 × 8	18	3	0 × 4	27			

TABLE XIV. DIFFICULTY OF THE DIVISION COMBINATIONS OF GROUP 1 AS DETERMINED BY THE NUMBER OF TIMES THE COMBINATIONS WERE INCORRECTLY SOLVED

Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed
1	4/2	3	30	45/5	15	26	3/1	19
2	25/5	5	46	45/5	15	39	0/8	19
16	20/5	9	12	14/2	16	43	0/1	19
3	12/2	11	14	16/2	16	5	10/5	20
4	10/2	11	28	32/8	16	21	6/1	20
40	24/6	11	31	40/5	16	44	35/7	20
20	15/3	12	36	5/1	16	49	81/9	20
22	0/6	12	48	32/8	16	6	9/3	21
27	81/9	12	7	6/2	17	38	35/5	21
32	20/4	12	11	16/4	17	47	0/7	21
34	21/3	13	37	18/3	17	17	12/3	22
35	12/4	13	41	30/5	17	33	7/7	23
47	36/6	13	13	14/7	18	9	12/6	25
10	15/5	14	19	8/4	18	23	18/9	26
15	21/7	14	24	6/3	18	45	4/1	28
18	16/8	15	25	9/1	18	50	3/1	34
24	36/6	15	8	8/2	19			

number of times the combination was missed, shows that the two are far from agreement (Table XIII). Many of the combinations were not presented in what these 77 pupils found to be the order of difficulty. A number of the combinations in G2 should be listed in G1 and a like

number in G1 moved to G2 if the combinations are to be arranged in the order of difficulty for the pupils studied in this investigation.

The most difficult of the multiplication combinations were those which involved zero. In the G1 multiplication facts there were many

TABLE XV. DIFFICULTY OF THE DIVISION COMBINATIONS OF GROUP 2 AS DETERMINED BY THE NUMBER OF TIMES THE COMBINATIONS WERE INCORRECTLY SOLVED

Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed	Problem Number	Problem	Times Missed
1	30/6	1	7	0/5	22	24	49/7	28
5	27/9	8	8	9/9	22	30	54/6	28
11	40/8	13	13	2/2	22	40	27/3	28
9	8/1	14	14	18/2	22	42	54/9	28
10	7/1	14	27	63/9	23	38	24/3	29
12	72/9	14	28	36/4	23	44	48/8	29
21	32/4	14	31	6/6	23	48	36/4	29
2	0/2	15	20	3/3	24	23	5/5	30
4	0/9	16	35	42/7	24	46	54/6	30
3	0/3	19	36	48/6	24	29	1/1	31
18	45/9	19	16	18/6	25	43	28/4	32
25	72/8	19	17	4/4	25	49	63/9	32
26	56/8	20	19	8/8	26	32	2/1	33
6	0/4	21	33	28/7	26	41	24/8	34
15	24/4	21	34	56/7	26	45	64/8	35
37	36/9	21	39	63/7	26	50	56/8	36
47	1/1	21	22	42/6	28			

incorrect solutions in combinations such as 5×0 and 2×0. In general, the combinations of the G1 problems, with the exception of those involving zero, seemed to be sufficiently well mastered. Zero times a digit was also the most difficult type of problem in the G2 combinations. It seems reasonable to conclude from the data concerning the number of times each combination was missed by the 77 pupils in the investigation that the combinations containing the larger digits which were found in the G2 problems were not known as well as they should have been by the subjects.

The number of times each combination in G1 of the division problems was missed is given in Table XIV. The range of frequencies for the incorrect solution is 3 to 34. Problem 1 ($4 \div 2$) was missed 3 times and was therefore the easiest combination. Problem 50 ($3 \div 1$) was missed 34 times and was the most difficult combination of the G1 problems. From the data in Table XIV, it is noted that there is little uniformity between the order of presentation of the combinations and the order of difficulty as determined in this study.

The number of times each combination in G2 of the division problems was incorrectly solved is given in Table XV. The range of frequencies of incorrect solutions is 1 to 36. Problem 1 ($30 \div 6$) was missed only once and is therefore the easiest combination in the G2 problems. It will be noted that problem 1 of the G2 combinations was missed less frequently than problem 1 of the G1 series. Problem 50 ($56 \div 8$), missed 36 times, was the most difficult combination in the series. Although there is considerable variation in the order of difficulty of the G2 division basic facts from the order of presentation, there is more correspondence than was found in the G1 problems.

The most difficult combinations of the G1 division problems were combinations in which a digit was divided by 1. The combinations containing zero and digits divided by the same digit also were missed frequently. The high frequency of the incorrect solutions of the G1 division combinations indicates that the subjects had had insufficient drill in division. None of the larger combinations of the G2 division problems were sufficiently mastered by the group. As in the G1 problems, those combinations containing zero as a dividend and those containing the larger digits were the ones most frequently missed.

The order of difficulty of the combinations in the four fundamental processes found in this study varied considerably from that found in other studies. There are at least three possible explanations for this situation: (1) the method of presentation (in this study the number master was used); (2) the length of time allowed to solve the combinations (the method of presentation provided a constant and limited time for the solution of each problem); and (3) the training of the pupils (the difficulty or lack of difficulty of a combination may not depend upon the digits in the combination but may be a function of the experience the pupil has had with that combination).

Further evidence that the order of presentation of the combinations was not the order of difficulty for the fourth grade pupils in this investigation may be obtained by examining the total number of times

each combination was incorrectly solved. These data are given in Table XVI. The total scores were computed by adding the number of times each G1 combination was missed to the number of times the same numbered G2 combination was incorrectly solved. In addition, for example, combination 1, G1, was missed once and combination 1, G2, was missed four times, making a total of five incorrect solutions for problems 1, G1 and G2.

The data in Table XVI are plotted in the curves in Figure 6. In examining this figure it is noted that fewer combinations were missed

TABLE XVI. SUM OF THE NUMBER OF TIMES EACH G1 AND G2 COMBINATION WAS INCORRECTLY SOLVED

Problem	Addition	Subtraction	Multiplication	Division
1	5	9	29	4
2	4	16	16	23
3	6	19	29	30
4	6	17	12	27
5	5	20	17	28
6	7	11	27	42
7	0	11	28	39
8	10	19	12	41
9	11	17	16	39
10	16	18	32	28
11	13	13	30	30
12	11	16	31	30
13	8	17	17	40
14	8	14	12	38
15	8	18	18	35
16	6	20	33	34
17	9	30	35	47
18	10	29	34	34
19	6	24	28	44
20	8	18	35	36
21	6	25	35	34
22	9	19	21	30
23	10	18	19	56
24	10	19	19	46
25	9	22	32	37
26	5	15	15	39
27	9	18	12	35
28	9	18	15	39
29	9	26	21	46
30	19	20	24	43
31	6	21	18	40
32	8	23	27	45
33	10	33	25	49
34	15	36	26	39
35	11	39	23	37
36	11	31	30	37
37	15	36	31	38
38	17	28	29	50
39	16	46	46	44
40	21	42	33	39
41	17	46	41	51
42	16	34	55	48
43	16	45	29	51
44	17	40	32	49
45	22	42	34	63
46	24	39	51	45
47	24	45	43	35
48	21	39	48	45
49	29	45	37	52
50	36	53	59	70

in addition than in subtraction, multiplication, or division; that there was not much difference in the number of combinations missed in subtraction and multiplication, although there were more than in addition; and that more combinations were incorrectly solved in division than in any of the other fundamental processes.

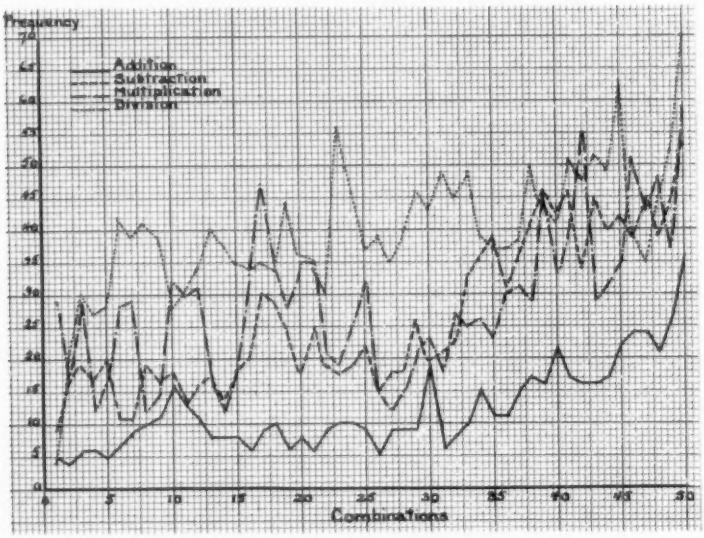


Fig. 6. Sum of the Number of Times Each G1 and G2 Combination Was Incorrectly Solved.

A study of the relative difficulty of the number combinations as determined by this investigation indicates that the tables of relative difficulty of number combinations cannot be applied in teaching without modification. The difficulty of the number combinations varies considerably as one proceeds through the school grades. The relative difficulty in a particular grade will depend to a large degree on whether practice on all of the combinations has been distributed well or poorly in the preceding grade. If practice in the preceding grade was distributed according to the real difficulty of the combinations for each pupil, there is every reason to believe that the difficulty would be greatly equalized. Whether drill on the different combinations is equalized or not depends chiefly on the textbook and the practice materials used. The inequalities in the frequency of distribution of drill on the combinations have been indicated by Thorndike[20] and, in greater detail, by Clapp,[21] who gives a complete analysis of all the combinations employed in two textbooks and establishes a relation on the number of errors.

[20] Thorndike, Edward L. "The Psychology of Drill in Arithmetic: The Amount of Practice." *Journal of Educational Psychology* 12:183-94, April, 1921.

[21] Clapp, Frank L. *The Number Combinations: Their Relative Difficulty and the Frequency of Their Appearance in Text-Books.* University of Wisconsin, Bureau of Educational Research, Bulletins No. 1 and 2, Madison, 1924. 20 and 126 pp.

Other factors that affect the difficulty of number combinations are the form in which the combinations appear, that is, whether the combinations appear as a combination or in problems. Another factor is whether the time for the solution of the combinations is limited. In general, the more difficult combinations in the timed test, as measured by the number of incorrect solutions, are relatively less difficult in the untimed test, since in the latter case there is more opportunity to find a correct response to a combination which is not thoroughly automatized.

A study of the relative difficulty of the combinations in this investigation indicates clearly that a combination presented in direct and reversed fashion, as $7 + 9$ and $9 + 7$, may be very unequal in difficulty and that both forms should be taught. This is sufficient evidence to support the contention that pupils should be taught the 100 combinations rather than only 45.

CONCLUSIONS BASED ON GROUP DIAGNOSIS

Some of the more important results of the group diagnosis are the following:

1. As the combinations increase in size the number of errors increases.

2. Pupils exhibiting mastery of the combinations containing the smaller digits were also efficient in solving the higher combinations, but to a lesser extent.

3. The basic facts of addition were more thoroughly mastered than were those of subtraction, multiplication, and division. There was little difference in the mastery of the basic facts of subtraction and multiplication, and these combinations, although not as well known as those of addition, were better executed than those of division.

4. The less efficient the pupils were in the basic addition facts, the less able they were to execute the basic number facts in the other fundamental processes.

5. The combinations missed most frequently were those containing zero.

6. The difficulty of a combination was due in part to the digits and in part to the order of the digits in the combination.

7. The difficulty of a combination may be due to (1) method of presentation, (2) time given to solve the combination, and (3) the kind and amount of experience the pupil had previously had with the combination.

8. The order of difficulty of the combinations varied considerably from that found in other studies.

9. The tables of relative difficulty of the basic number combinations should be modified to meet the needs of the pupils.

10. The 100 basic number facts should be taught rather than the 45.

EXPERIMENTAL CONDITIONS FOR INDIVIDUAL DIAGNOSIS

From the results of the group diagnosis with the number master, the five pupils making the lowest scores in each of the four fundamental processes were selected for individual study.

Apparatus and materials.—The dictaphone was selected to record the number activity of the pupils in order to learn as accurately and as completely as possible their mental processes. The machine used was a standard instrument (Type A, Model 10X) manufactured by the Dictaphone Corporation of New York. No changes or modifications were made in the dictaphone.

The materials used in the individual diagnosis were the same as those used in the group study.[22]

Procedure.—The diagnosis was conducted in a quiet room free from distractions. Only the experimenter and one pupil were in the laboratory during a testing series. On the first day a subject reported, he was given a series of number combinations on paper in addition, subtraction, multiplication, and division, and he was instructed to speak his thoughts aloud while working the problems. Pencil and paper computations were continued until each subject worked the problems aloud without hesitation.

Following the pencil and paper computations, each subject was instructed in the use of the dictaphone and was given drill in speaking into the mouthpiece. Although the problems were on paper before the pupil, he was required to read each one aloud before giving the answer. Preliminary recording was continued for each subject until he spoke clearly and without hesitation. Since the situation was a novel one to the pupils, no great difficulty was experienced in training them to think aloud or to speak clearly and distinctly into the mouthpiece of the dictaphone.

Following the preliminary training, the 100 combinations in addition were given. The 100 combinations in subtraction, in multiplication, and in division followed in the order named. Each subject had only 100 combinations on any one day, and there was a rest period of 10 minutes at the end of the first 50 problems. The subject began each recording by dictating his name and the type of number facts with which he was working that day. The length of the testing period varied with each subject, since no instructions were given concerning speed and accuracy. Individual diagnoses for each pupil were made in only those fundamental processes in which the pupil had been one of the five least efficient.

During the recording the experimenter observed and recorded the physical behavior of the pupil. These recordings were later studied carefully. Records were run and re-run until all significant activities were discovered and listed. Physical behavior was analyzed and combined with the significant number activities. In that the resulting data do not lend themselves to statistical treatment, a general discussion of the nine pupils studied individually will be given, together with a summary of the diagnosis and a case history for each subject.

Subjects.—Only nine different subjects represent the five least efficient pupils in each of the four fundamental processes. The five least efficient pupils in addition were Subjects 2, 17, 35, 43, and 51; in subtraction, Subjects 2, 7, 17, 35, and 52; in multiplication, Subjects 7,

[22] See sample of problems on page 12.

17, 35, 48, and 52; and in division, Subjects 7, 17, 35, 41, and 48. Subjects 17 and 35 were among the five least efficient in all four of the fundamental processes. Subject 7 ranked among the five lowest in subtraction, multiplication, and division; Subjects 2, 48, and 51 were among the five least efficient in two of the fundamental processes (Subject 2 in addition and subtraction, Subject 48 in multiplication and division, and Subject 52 in subtraction and multiplication); Subject 41 was among the five least efficient only in division and Subjects 43 and 51 only in addition.

TABLE XVII. SCORES MADE BY THE NINE SUBJECTS OF THE INDIVIDUAL DIAGNOSIS GROUP IN EACH OF THE FOUR FUNDAMENTALS

Subject	Addition	Subtraction	Multiplication	Division
2	77	37	78	64
7	79	53	30	22
17	51	8	30	4
35	69	41	18	11
41	90	57	65	21
43	75	66	53	33
48	93	83	45	30
51	32	89	94	68
52	88	48	42	46

A summary of some of the significant information concerning the nine subjects is as follows:

Sex.—There were five boys, Subjects 7, 41, 43, 48, and 51, and four girls, Subjects 2, 17, 35, and 52.

Age.—The age range of the pupils was from 9 to 12. One pupil, Subject 43, was 9 years old; four pupils, Subjects 2, 41, 48, and 51, were 10; three pupils, Subjects 7, 17, and 52, were 11; and one pupil, Subject 35, was 12 years of age.

Health.—Of the nine pupils only one, Subject 17, was in excellent health. The health of Subject 52 was considered satisfactory but the other seven pupils were defective in one or more health factors.

Intelligence.—In relation to intelligence one pupil, Subject 45, had a high I.Q.; four pupils, Subjects 2, 41, 48, and 51, had I.Q.'s sufficiently high to class them as normal; and four pupils had definitely low I.Q.'s.

Home environment.—It is always difficult to evaluate the home life of a pupil, but the information concerning the nine pupils indicates that only one, Subject 43, came from a home environment that could be classified as good; the home environment of three pupils, Subjects 35, 41, and 51, seemed to be satisfactory. Poor financial status of the parents of four pupils, Subjects 2, 7, 17, and 48, tended to detract from the wholesomeness of their home life. The home environment of one pupil, Subject 52, was unsatisfactory because of the chronic illness of the mother.

School citizenship.—Only one pupil, Subject 43, was rated by his principal and teachers as having superior citizenship; four pupils, Subjects 2, 17, 35, and 52, were rated as satisfactory on conduct; three

pupils, Subjects 41, 48, and 51, were rated low on citizenship; and one pupil, Subject 7, was a definite "problem" case.

School achievement.—One pupil, Subject 43, was doing excellent work in school; four pupils, Subjects 17, 41, 48, and 52, were making satisfactory progress; and four pupils, Subjects 2, 7, 35, and 51, were making unsatisfactory progress.

Arithmetic achievement.—As measured by school marks, three of the nine pupils, Subjects 41, 43, and 51, were making satisfactory progress in arithmetic; the other six pupils, Subjects 2, 7, 17, 35, 48, and 52, were either failing or doing inferior work.

CASE HISTORIES AND RESULTS OF DIAGNOSIS

A discussion of the case history, score made on the group analysis, and results of the individual analysis for each of these nine subjects follows:

Subject 2.—Girl, born January 4, 1928. Her father and mother are living, but the family is poor financially. B—— is overweight, slow, and easy-going, and does not have a very cheerful disposition. She seems to have ear and eye trouble; otherwise her health is good. She did not object to taking the test, but was rather indifferent and uninterested.

She had made D in arithmetic for five semesters and made a poor showing on the arithmetic tests, but she gave the impression that if she had exerted more effort her record would have been better.

On tests of achievement she was just a little below her actual age. Her intelligence ratio was 91.

On the group analysis Subject 2 was particularly poor in subtraction, solving correctly only 37 of the 100 problems. She did better in addition, solving 77 of the 100 combinations correctly but, because the group as a whole made higher scores in addition than in the other problems, her score of 77 was not high, although it was not quite low enough to place her in the group of the five lowest. In comparison with the scores of the other members of the unselected group, her scores of 78 on multiplication and 64 on division indicate that she was also deficient in the basic facts of both these operations.

On the individual analysis, B—— was one of the five least efficient in both addition and subtraction. She began working the addition combinations very rapidly, but this fast rate did not continue. She frequently read a problem, then turned and looked out of the window. As far as the experimenter could discover her mental processes were a complete blank during this period of shift of attention.

In the first 50 problems she made only 8 errors. The problems she missed were multiplied rather than added. The fact that she executed correctly 42 of the 50 combinations does not mean that she knew them. She knew very few, but arrived at the other correct answers by counting. She frequently counted orally and at the same time tapped either her pencil or her toe, sometimes both. She was very slow, often taking as much as a minute to arrive at the answer for a simple combination.

For the second group of 50 problems, B—— used about twice as much time as for the first. She continued counting, but constant shifting of her focus of attention made even counting difficult. In this group 16 answers were incorrect.

A few special problems were interesting to note. Between the reading of the two problems, $\begin{array}{r} 8 \\ +\,4 \\ \hline 12 \end{array}$ and $\begin{array}{r} 4 \\ +\,8 \\ \hline 12 \end{array}$, B—— hesitated a long time. She seemed to recognize the similarity of the problems and was careful to read them in the same order. The sum of $9 + 8$ was given as 19 in two different places. Likewise the sum of $7 + 6$ was given as 12 two different times, but in both cases was changed to 13.

B——was even slower in working the subtraction problems than she had been in the addition combinations. Frequently, whenever a number was subtracted from itself that number was given as the answer. Other than this type, the 27 errors in the first 50 problems were of no particular kind.

In the second group of 50 subtraction problems there were 28 incorrect answers. Frequently the answer given was the number subtracted. In one problem, $\begin{array}{r} 16 \\ -\,8 \\ \hline \end{array}$, she first read "8 from 6," hesitated a long time, then read the problem correctly, but her attention shifted before she had completed counting from 8 to 16 and an incorrect answer was given.

Subject 7.—Boy, born January 9, 1927. His father is a day laborer and is rather poor financially. There are three girls in the family, one younger in school, one in high school, and one who is grown. The only other boy of the family was killed three years ago.

L—— is considered "the problem of Hunter School." The teachers rate him as worse than mischievous. He bothers other children and is always getting into scrapes. It is quite probable that his parents have spoiled him since he is the only boy of the family.

He remained in the 1A grade for three semesters. In his arithmetic record he has made C once, D twice, and F twice. Last Christmas he had the measles, which left him with a weak heart. This semester he has been indifferent and unattentive. In taking the test he was unusually slow. After long hesitation he would usually give the wrong answer. He was much interested in the dictaphone. He asked a number of questions about the mechanics of it, and hoped he could make more records some day.

Previous tests showed an intelligence quotient of 85, and his achievement was somewhat below his actual capacity.

On the group analysis Subject 7, with scores of 53 in subtraction, 30 in multiplication, and 22 in division, exhibited little knowledge of the basic facts of these processes. His score of 79 in addition was just sufficiently high to keep him out of the five lowest in the basic addition facts.

On the individual analysis he was exceptionally slow in arriving at his answers, and after his laborious process of finding the answer

he would decide it was incorrect and go through the entire procedure again. In all but the simplest multiplication combinations, such as 2×1 and 2×2, he had not the slightest idea of what the correct product should be; neither did he know any method which he could use in solving the problems. In combinations such as 3×7, 4×8, etc., he attempted to add $7 + 7 + 7$ or $8 + 8 + 8 + 8$. He did this orally, counting on his fingers at the same time. His answers were nearly always incorrect, as his attention would shift before he had completed the addition. The order of multiplication bothered him. If 0 was written below any number, the answer was given as that number, but if a number was written below 0, the answer was given as 0. Of the second 50 problems, 38 were incorrect.

L—— had much trouble in arriving at an answer to the division problems. It took him about three times as long to work a division problem as it did to work a multiplication combination. He knew a few of the simple division problems, but he had no idea as to how to solve the rest. Frequently he arrived at an answer orally and then wrote down a different answer on his paper. Of the second 50 problems, 41 were incorrect.

L—— guessed at many of the answers. He read the problem, squirmed around in his chair, looked at the experimenter, mumbled a few numbers, then distinctly spoke a number and recorded it or some other number on his paper. In the last 50 problems he gave 7 as an answer 15 times.

Subject 17.—Girl, born February 17, 1927. Her father is a day laborer and makes a rather poor living. A sister is attending high school at present.

A—— entered school the first semester of 1933-34 and was retained in 1B one semester. Her grades, with the exception of this semester, have ranged from 75 to 85. Since the school has changed to letter grades she has made mostly C's, with a few D's and F's. During the first semester of 1937-38 she made eight C's and one F in the 4B grade. This F was in arithmetic.

A—— has failed in arithmetic for five semesters and has made D's all of the other semesters. She does well in reading and geography, but it is a general feeling among teachers that she will never learn any arithmetic. On the present test only a very few answers were given correctly and those seemed to be accidental rather than the result of any understanding of the problem.

She is in excellent health and is sturdy and chubby. She has a pleasant disposition and seemed to enjoy taking the tests.

An intelligence test given in the first grade showed her I.Q. to be 76.

On the group analysis Subject 17, with scores of 51 in addition, 8 in subtraction, 30 in multiplication, and 4 in division, showed an almost entire lack of of knowledge of the number combinations in the four fundamentals. Her highest score was in addition, but she solved only 9 combinations correctly in the second 50 problems.

A—— is a good student in her other subjects, especially in those involving considerable reading. On the individual analysis she read the addition problems rapidly and correctly, giving an answer immediately, but her answers were usually wrong. She did not count or tap. Answers came so quickly after reading the problems that if her answers had not been checked one would have assumed that she knew the combinations. There were, however, certain significant characteristics of her work in addition. She always obtained the correct answer when one (1) was added to another number. When two (2) was added to a number, her answer was that number increased by one. In the first 50 problems, 25 of her answers were correct. In the second group, only seven of her answers were correct. In this second group of 50, 8 was given for an answer 23 times. A—— knew only the simplest number combinations and she possessed none of the roundabout methods of acquiring correct answers. Therefore, she was forced to guess.

She worked with equal rapidity in the subtraction problems. In the first 50 problems the only ones that were solved correctly were those where zero was to be subtracted from a number. When one (1) was to be subtracted from any number that number was invariably given as the answer. As in addition, the number 8 was a favorite answer. It occurred 17 times in the second group of problems. A—— knew even less about the basic combinations in subtraction than she did in addition. It is doubtful whether she was able to comprehend the difference between subtraction and addition. Apparently all problems to her were addition problems.

In the multiplication problems, A—— continued to add rather than to multiply, and she read many of the multiplication problems incorrectly. Her answers were pure guesses or they were approximate answers arrived at by addition. In the second set of problems with the larger numbers, she always gave larger answers, but none of them were correct. Such numbers as 82, 72, 64, and 55 occurred quite frequently as answers in the last 50 problems.

There is little to say about A——'s division. The only problems she seemed to understand as being division problems were those in which a number was to be divided by one (1). For the others she would read the problem and immediately give an answer which was purely a guess.

With A—— it was not faulty habit that reduced her efficiency in the four fundamental processes, but it was an absolute lack of knowledge of the basic number combinations. The average first grade pupil would have greater number knowledge than she.

Subject 35.—Girl, 12 years of age, small and thin. She has one brother. Her father is dead and her mother, an immoral character, receives a pension. This girl, like her mother, takes treatments for syphilis every week. Her teeth are bad, and she was withdrawn from school one semester because of her eyes, which squint. She wears glasses. Her muscular coördination is poor.

Mentally J—— is inferior. Her I.Q. is approximately 80. She spent five semesters in the first grade and has been making F in arithmetic this semester. A recent test in reading shows her to be achieving beyond her capacity at present.

The child is cheerful and is termed a "live wire." However, she does not stay with anything very long at a time. In view of her mentality and physical condition it is not surprising that her scores in arithmetic were low.

J——, with scores of 69 in addition, 41 in subtraction, 18 in multiplication, and 11 in division on the group analysis, exhibited little efficiency in the basic number facts in the four processes. She ranked as one of the five least efficient on all four types of problems.

On the individual analysis she read the addition problems rapidly, recognizing the simple combinations and giving the correct answers. A number of times she gave the correct answer orally but put an incorrect answer on her paper. She failed to recognize the similarity of two problems when the digits were given in reverse order. On the second list of problems she frequently hesitated between the reading of the problem and the statement of the answer. During this period of hesitation, although there were no sounds, she was counting on her fingers. She attempted to do the counting very rapidly and therefore most of her answers were incorrect. In the second list of problems, 39 of the 50 solutions were incorrect. She did not know the 100 addition combinations, but she did know how to get their correct answers by counting.

J—— worked rapidly on the first 50 problems in subtraction. She understood that subtraction meant a different process from that of addition. She read the problem and immediately gave the answer. However, 24 of the first 50 were incorrect. When a number was subtracted from the same number the answer was invariably given as that number. One interesting characteristic of her answers was that in practically all cases the incorrect answer was only one digit from the correct answer.

On the second group of subtraction problems, J—— allowed considerable time to elapse between the reading of the problem and the giving of the answer. During this period her lips moved but she made no sound. Frequently she repeated the problem several times. Recordings and observations by the experimenter are in agreement that, in all but the simplest of the subtraction problems, J—— was guessing at the answer. Of the second set of 50 problems, 36 were incorrect.

In multiplication J—— said she kept thinking about working problems in subtraction. She worked rapidly, but only half of the first 50 problems were correct. In the second set of problems, with the larger numbers, she guessed larger answers.

There is very little to say concerning J——'s performance in division as she had little or no conception of how to arrive at the correct answer. She read the problems and immediately gave an answer. Of the 100 division problems, only four answers were correct. There was no particular type of difficulty, just the fact that she did not know how to divide and there was no method that she knew by which she could arrive at the correct solution.

Subject 41.—Boy, born December 24, 1928. His father is a "meter man." F——'s home life is ordinary, with a fair financial status. He has one baby brother.

He entered school February 4, 1935, and during the summer of 1937 he attended the University elementary school. His grades in arithmetic have been C's for four semesters. On the Stanford Achievement Test he ranked higher in arithmetic reasoning and comprehension than in paragraph and word meaning and dictation. On two tests he ranked normal in I.Q.

F—— is a very apathetic, slow, and listless child. He wears glasses and probably is not very strong. His mother is quite intelligent. One of his teachers stated that F—— lacked aggressiveness, but that he is improving, although he remains indifferent and obstinate at times. He has a negative attitude and has probably been pampered by his parents. He is considered nonsocial and babyish. He needs more individual help.

F—— did satisfactory work on the addition combinations on the group analysis, making a score of 90. He was poor in subtraction, multiplication, and division, scoring respectively 57, 65, and 21. These scores indicate that he did not know the basic number facts of subtraction, multiplication, or division, although he was among the five least efficient only in division.

On the individual analysis F—— read the division problems with much hesitation and gave two or three answers to each problem, finally putting one down on his paper and stating in the dictaphone that this was the correct answer. Of the first 50 problems, however, only 23 were correct. With the larger answers in the second set of problems, F—— was lost. He sometimes gave as many as six answers, stating after each of the first five "This answer is not right," and finally writing the sixth answer on his paper. Fifteen of the second set of problems were correct. F—— had the habit of reading a problem two or three times, then passing on to some other problem without giving the answer to the first problem he read.

On the last 25 problems of the second set, F—— apparently made no attempt to arrive at a correct solution. He read the problem through two or three times and then gave an answer. No correct answers were given for these last 25 problems. The number six (6) occurred most frequently in his answers. F—— did not know the basic division facts and he knew no method by which he could arrive at a correct solution.

Subject 43.—Boy, 9 years of age. D—— is very likable and attractive, one of the most interesting of the cases. He comes from a good family, which takes much interest in his school work. His father operates a filling station and his mother is an active P.T.A. worker. There is a younger child, a girl, in the family.

During the testing the boy spoke of his bad nerves. His mother, who later heard his recording, stated that according to the doctor, he was "one step removed from St. Vitus dance." He held the paper, on which the problems were typed, close to his eyes and there was a tendency to squint, indicating the probability that his eyesight is weak. His heart likewise is poor. His tonsils are to be removed this summer in an effort to improve his health.

He is considered an excellent student. A few years ago he was tested for intelligence and had an I.Q. of 128. On reading tests he ranks approximately two grades in advance of his present grade. Achievement tests show that he has a performance age of almost a year more than his chronological age.

D—— was ridiculed by his classmates for being slow. This slowness was probably largely responsible for his poor scores on the fundamentals. His grades in arithmetic have been three B's and one C.

D——, with scores of 75 in addition, 66 in subtraction, 53 in multiplication, and 33 in division on the group analysis, exhibited little efficiency in the basic number facts although he also was among the five poorest only in addition.

On the individual analysis, with the problems before him on paper and as much time as he desired, D—— gave correctly every solution to the 100 addition problems while reading them over the dictaphone. He appeared very nervous at first, spoke loud and fast, and hesitated a great deal on particular problems, such as $\begin{array}{r} 9 \\ +4 \\ \hline 13 \end{array}$, $\begin{array}{r} 9 \\ +5 \\ \hline 14 \end{array}$, $\begin{array}{r} 9 \\ +6 \\ \hline 15 \end{array}$, $\begin{array}{r} 5 \\ +8 \\ \hline 13 \end{array}$. Two problems with the same digits written in reverse order caused hesitation. The second was read like the first, then changed.

When D—— did not recognize the answer, he counted. Counting was subvocal, but the movement of the lips could be observed. Several times he looked at the combination for a few seconds, then started to count, but he remembered the correct answer before completing the counting.

From the results of the dictaphone records and the experimenter's observations there seem to be two reasons why D—— did poorly on the number master: (1) he was a slow worker and, although he knew how to arrive at the correct solution in addition by counting, he did not have time to count in the group diagnosis; and (2) his vision seemed to be poor and this handicapped him in reading the problems rapidly as they were exhibited.

Subject 48.—Boy, born September 10, 1928. His father is poorly educated and is not employed regularly. One child in the family is dead. B—— seems to have good health. He may, however, have some trouble with his eyes.

B—— is considered mischievous by his teachers. He was quite interested in the dictaphone and made a special effort to make his second record louder after hearing himself on the first.

According to a Binet test, his I.Q. is 97. His recent grades in arithmetic have been two C's and one D, his actual achievement being somewhat below his capacity.

B——, with scores of 93 in addition and 83 in subtraction on the group analysis, did satisfactory work, but his scores of 45 in multiplication and 30 in division indicated that he was not familiar with the basic number facts in these processes.

On the individual analysis, B—— seemed to have a knowledge of most of the simpler multiplication combinations. This fact is borne out

by the fact that the first 36 problems were given without error. He recognized the fact that the order of multiplication made no difference. On one special type of problem he always gave an incorrect answer. When zero was multiplied by any number, that number was always given as the answer. A number of other combinations had been learned incorrectly. The solutions, $\begin{array}{r} 3 \\ \times 4 \\ \hline 18 \end{array}$, and $\begin{array}{r} 4 \\ \times 3 \\ \hline 18 \end{array}$, were given without hesitation.

That B—— was thinking rather than just writing the first thing that came into his mind was shown by the number of corrections he made. Frequently, after reading and writing a wrong answer, he corrected it. He seemed to be more conscious of a written mistake than of one which was merely spoken. He read some problems wrong but gave the correct answer. For one problem, $\begin{array}{r} 3 \\ \times 3 \\ \hline 9 \end{array}$, he said and wrote 12 for the answer, then changed it to 9 on paper but did not read the correction.

As the problems in the last group of 50 grew harder, B——'s difficulties increased. He read the answers more slowly and made many more errors. Problems involving digits of 6, 7, 8, and 9 were always given wrong. In this group, 39 answers were incorrect. Of these answers it is interesting to note that there was a very noticeable tendency to give an answer ending in 5 or 0. Another interesting observation was that, of these 39 incorrect answers, all except 5 were answers containing two digits. B—— had not learned the more difficult multiplication combinations and he had no method by which he could arrive at a correct solution.

Although B—— answered the harder combinations more slowly than the easier ones, he did not hesitate on these problems as much as he did on the simpler ones. He was aware of the fact that he could not think them out. The answers to two problems involving the same digits, $\begin{array}{r} 6 \\ \times 7 \end{array}$ and $\begin{array}{r} 7 \\ \times 6 \end{array}$, coming one after the other, were given as 28 and 50 without hesitation.

For B——, division problems caused more difficulties than multiplication. His rate for reading the problem and giving the answer was fairly constant. He did not stop to think. Here again zero gave him trouble. Usually when a digit was divided by 1 the answer was given as one less than the digit. Now and then he read the problem in reverse order, as 5 divided by 25, but gave the correct answer.

He had little knowledge of the basic division facts—33 of the first 50 problems and 44 of the second 50 were answered incorrectly. The problems given the correct answers were not necessarily those containing the smallest digits. The numbers 4 and 5 were given as answers more frequently than any others.

Subject 51.—Boy, 10 years of age. His father is a chauffeur. The child seems to be healthy although he probably has weak eyes. He was retarded one semester in the 2B grade. Mentally, he is normal.

E——'s teachers state that he does not apply himself, and this is borne out by test results which show that he is not achieveing up to his capacity. The Stanford Achievement Test reveals that in arithmetic reasoning and comprehension, however, he not only ranks much higher than in his other subjects but is more than a grade superior. His marks in arithmetic during the regular school years have been four B's and one C. In the light of these results we would not have expected him to be one of the poorest students in arithmetic fundamentals.

E——, with a score of 32 in addition on the group analysis, was the least efficient of the group of 77. His scores of 89 in subtraction and 94 in multiplication indicated that he was familiar with the combinations in these processes, but his score of 68 in division showed a lack of efficiency in this ability.

On the individual analysis, E—— was exceptionally slow in arriving at his answers. He read the problem, usually the smallest digit first, and immediately gave the answer. However, he did not record the answer, but would say, "No, I don't believe that answer is right." Then he gave another answer without re-reading his problem and, finally, he counted to get a correct solution.

Combinations containing zero were beyond his comprehension, as he could not obtain the answer by counting. His answers to all such problems were zero. There was much overt physical motion connected with the solving of the number combinations, but, with the exception of lip movements when he was not counting aloud, there seemed to be no definite connection between the physical behavior and the attempt to solve the problem.

E——'s periods of concentration were short. If he was not satisfied with the accuracy of the answer after giving two or three answers to a combination, he would turn, look out of the window, and forget about the problem. Frequently minutes elapsed before attention was again directed toward the solution of the problem.

For some of the combinations, especially those in the second list, containing the larger digits, E—— did not attempt to find a solution. He guessed at an answer or said he did not know. After he had decided he did not know, that was the end of attention for that combination. In the first 50 combinations E—— made 15 incorrect answers. Nine of the 15 incorrect solutions involved combinations containing zero, 3 were multiplied instead of added, and 3 were missed because of shift of attention during counting. In the second list of combinations, 20 answers were incorrect. Again all combinations containing zero were incorrectly solved. Faulty counting or guessing accounted for the other errors.

E—— had two serious deficiencies: (1) he did not know the basic number facts in addition, and (2) his attention span was so short that, although he knew how to arrive at the correct answer by counting, he usually made mistakes.

Subject 52.—Girl, 11 years old, calm and quiet. She appears more confident and steady than one would expect of a child her age. Her mother has been ill for some time and this may be instrumental in

developing her assurance and poise. M—— possesses a pleasant and cheerful disposition and, although her school achievement is not high, she seems to enjoy her work. She was very coöperative and asked the experimenter if she might help score her papers.

Her work in arithmetic has been poor. Results on an intelligence test show that she has an I.Q. of 74. According to a reading test which she was given recently, she is achieving up to her capacity.

On the group analysis, M—— showed little knowledge of the basic number facts in any of the fundamental processes, except in those of addition. She made scores of 88 in addition, 48 in subtraction, 42 in multiplication, and 46 in division, and was among the five least efficient in subtraction and multiplication.

On the individual analysis, M——'s difficulties in subtraction were hard to analyze. It seems unusual that, when a number was to be subtracted from itself, she always gave the answer as one less than that number. Also, when any digit was to be subtracted from 10, the problem was multiplied. These calculations were executed orally, but there was no explanation of why such procedures were followed.

Of the first 50 problems, 29 were incorrect, and of the second 50, 39 were incorrect. Five problems were passed over with no attempt to give an answer. There was much overt physical activity, such as squirming in the chair, constant shifting of feet and hands, and twisting of head, but there seemed to be no connection between the physical activity and the attempt to solve the problems.

M—— may be complimented on the fact that zero gave her no trouble. Nor were her difficulties necessarily with the problems containing the largest digits. In the multiplication tests she was careful always to read the problems in the same order. In the first list of 50 there were 12 problems for which she stated that she did not know the answer. Of the remaining 38, only three answers were incorrect. The twelfth problem was the first to give trouble. She asked what to do about it and was instructed to try to solve the problem.

In the second group of multiplication problems M—— read every problem as if it were subtraction, but she did not give the answer as such. To 27 problems she stated that she did not know the answer and to 5 others she gave an incorrect answer. After the 27 problems, she seemed to lose confidence in herself and did not take time to think. She gave only 3 more answers and these were not correct.

It was interesting to note that the incorrect answers were usually connected by some process with the factors of the problem. It was evident that M—— did not know the basic facts of subtraction and multiplication, for, even when she had unlimited time, she could not solve the problems correctly. Her behavior indicated that she did not understand the difference in procedure between subtraction and multiplication.

CONCLUSIONS BASED ON INDIVIDUAL DIAGNOSIS

The nature of the data resulting from the individual diagnosis precludes the possibility of stating specific conclusions which are com-

mon to all of the nine pupils. The results of an individual diagnosis are pertinent only to the individual studied, and the information secured should not be considered as applying equally well to other individuals of like efficiency. A study of the results of the individual study does, however, reveal the following interesting general information concerning the pupils and their performance:

1. Numerous health defects.
2. Poor home environment.
3. Low level of school citizenship.
4. Inferior school achievement.
5. Low achievement in arithmetic.
6. Low intelligence.
7. Inability to concentrate—short attention span.
8. Lack of interest and indifference to number work.
9. Lack of knowledge of the basic number facts.
10. Inability to distinguish between the procedures to be followed in computing combinations in addition, subtraction, multiplication, and division.
11. Counting, tapping, and other types of overt physical activity.
12. Much guessing of answers.
13. Lack of number sense.
14. Failure to recognize the similarity of two combinations when the digits were given in reverse order.
15. Much difficulty with combinations containing zero.

GENERAL SUMMARY

The results obtained in this study provide ample proof of the value of using a group diagnosis in measuring the efficiency of fourth grade pupils in the basic number facts of arithmetic. The objectivity, the accuracy, and the speed of diagnosis makes the number master a helpful and desirable device for the teacher. Group diagnosis, followed by an individual study of those pupils deviating most widely from the norms of their group, provides the teacher with a picture of the mental processes of her pupils. This should lead to great improvement in the teaching of arithmetic. Individual study through the recording of the pupil's mental processes provides opportunity for careful analysis and the accurate measurement of remedial work.

In the planning and organization of drill work, the teacher needs to know the number facts with which her pupils are experiencing the most difficulty. She also should know whether the pupils are mastering the number facts of the difficult arithmetical processes with equal efficiency. The difficulty of zero combinations and reverse combinations and type errors must be discovered and provided for in the drill exercises if pupils are to progress satisfactorily in their number work. Such information was quickly and accurately secured in this investigation.

It is generally agreed that pupils should know the basic number facts of the four fundamental processes, yet, of the 77 fourth grade

pupils studied, approximately one-fourth of the group possessed an inadequate knowledge of the addition number facts; one-half the group had not satisfactorily mastered the number facts of subtraction and multiplication; and nearly three-fourths of the pupils exhibited low efficiency in the number facts of division. Having this information, the teacher will be better able to plan her drill exercises in relation to the individual needs of the pupils.

The results of the investigation have demonstrated the need of considering reaction time in studying the efficiency of fourth grade pupils in the basic number facts of the fundamental processes of arithmetic. Many pupils who were thought by the teacher to be making satisfactory progress in their number work were found to be computing the answer to number combinations by counting or by other inefficient methods. Information of this type is difficult to secure in situations where the pupil has unlimited time to execute the combinations.

The close relation between efficiency in the number facts of addition and those of subtraction, multiplication, and division found in this study indicates the futility of attempting to teach the number facts of subtraction, multiplication, and division until the basic facts of addition have been mastered by each pupil.

Difficulty with the number facts of the four fundamental processes of arithmetic may result from a number of widely different causes, as was shown by the individual diagnosis. Through individual diagnosis the teacher may discover the basic causes for inefficiency in number work, and, if each pupil's particular weaknesses are known, remedial exercises may be adapted to the individual needs of the pupils.

Teaching the Number Facts of Addition

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Teaching the Number Facts of Addition

By

HENRY LESTER SMITH and MERRILL T. EATON

INTRODUCTION

THE number of addition facts which should be taught in order to attain the highest level of efficiency in problem computation is a debatable question. The increase in facility in computing problems in subtraction, multiplication, and division that can be derived from drill in the higher decade number facts of addition is also a controversial question. All teachers of arithmetic believe that certain of the basic number facts of addition should be taught, but they do not agree on how many of these facts should be taught.

The ten digits, 0 to 9 inclusive, may be arranged in 100 combinations. These 100 include all of the zero combinations such as $0 + 1$ and all reverse combinations such as $3 + 4$ and $4 + 3$. Teachers of arithmetic believe that the pupils should know the 100 combinations, but some believe the zero-digit combinations and the reverse combinations may be learned without actually teaching them. There seems to be sufficient evidence at the present time, however, to justify the teaching of both the zero and the reverse combinations. A number of studies have shown that combinations containing zero are among those that are often incorrectly solved. These studies have also shown that the fact that a child knows the combination $6 + 9$ is no assurance that he knows the sum of $9 + 6$.

There is less evidence in relation to the advisability of teaching the combinations of two-place numbers with one-place numbers such as $7 + 24$ and $5 + 16$. There are 765 such combinations the sum of which is less than 100 and 1,000 combinations which may be formed from the numbers 0 to 99. Few teachers have advocated the teaching of all these facts, but some have thought that as many as 350 of the combinations should be taught. All these number facts should be known, but if transfer of learning may be relied upon, then it is unnecessary to spend time in learning all of them.

In considering the question of teaching the 45, the 100, the 350, the 500, the 765, or the 1,000 addition facts, teachers of arithmetic are faced with the problem of the results to be accomplished and the proper distribution of time to be devoted to the study of the basic facts of subtraction, multiplication, and division. If the execution of the number facts of addition is basic for the efficient execution of problems in subtraction, multiplication, and division, as some writers on methods of teaching arithmetic believe, then at least all the 1,000 addition facts should be taught. Surely, the best teaching procedures in this field cannot be developed until the value of teaching the higher decade combinations has been determined.

Some students of arithmetic consider the learning of number facts

to be similar to learning a language. If one may compare the learning of number facts with the learning of words, there seems to be no more reason to expect the pupil to know that $9 + 5$ is 14 after he has learned that $5 + 9$ is 14 than there is to expect him, after he has learned that the combination of letters a-r-m spells "arm," to reason that the same letters rearranged will spell "ram." Such comparisons may also be made between column addition and the organization of words into sentences. It hardly seems reasonable to expect that the child, after learning that $9 + 5$ is 14, will recognize in column addition that $9 + 15$ equals 24. It is more likely that $9 + 15$ will be $9 + 5$ and 1 to carry. This is the method that has often been used in column addition, and, as a result of a lack of knowledge of the higher combinations, speed has been greatly reduced and inaccuracies have increased.

The learning of number combinations, the language of arithmetic, is considered by many teachers to be a more difficult process than the learning of the child's native language. From the time the child is born he hears words and phrases and he enters school with a fairly large vocabulary. In addition, as rapidly as new words and ways of using them are learned in the classroom, the child has ample opportunity to use the new knowledge in his out-of-school life. This is less true of number facts. The child enters school with little knowledge of number facts, and there is little opportunity outside the schoolroom to use such facts as are learned in school. Then too, highly efficient methods of teaching language have been derived, but arithmetic is taught today very much as it was 25 years ago. The methods of selecting elementary teachers of arithmetic have been criticized on the ground that the teachers are usually chosen on the basis of their ability to do computative work rather than on the basis of their ability to teach children the language of arithmetic.

It has been argued that it is more difficult to attach meanings to numbers than to words. Some students claim there is more variability in the way numbers appear than in the way words appear. They maintain that letters forming the word "cat" always appear in much the same form, while the combination 7 plus 8 may appear as $7 + 8$, $\begin{array}{r} 8 \\ +\,7 \end{array}$, or $\begin{array}{r} 7 \\ +\,8 \end{array}$. Since an individual might be drilled on the a,b,c's for a lifetime and never learn words, it seems reasonable to conclude that he might be drilled on a few of the number facts of addition and never learn column addition.

There is little uniformity in textbooks in relation to the number of addition number facts they present for the pupil to learn, and frequently the amount of drill given each combination is not based on the difficulty of the combination as experienced by the pupil. Textbook writers have also failed to agree on whether the emphasis should be placed on speed or on accuracy. A more recent point of view concerning speed and accuracy is that they are two different functions. Accuracy is the result of arriving at a correct answer to a combination, and speed depends upon the method utilized in computing the answer. An accurate answer may be obtained from $8 + 7$ by counting, but speed may be secured only by knowing the complete pattern $8 + 7 = 15$.

There is sufficient evidence to show that increasing speed in addition does not increase accuracy. Neither does reducing speed below the individual's optimum rate of addition increase accuracy.

Because of the major importance of the questions concerning the extent to which the number facts of addition should be taught, this study proposes to measure: (1) changes in column addition resulting from drill on the basic number facts of addition; and (2) the transfer value of drill in the number facts of addition when computing problems in subtraction, multiplication, and division.

EXPERIMENTAL CONDITIONS

The experimental conditions for this investigation are as follows:

Subjects.—Twenty-three unselected college students were used as subjects. Their age range was from 20 to 25 years. Eighteen of the subjects were women and five were men.

25	38	49	52	63	74	86	97	80	42	30	94	76	85	01	84	75	62	53	72
82	52	34	36	78	92	37	54	62	35	17	36	81	92	31	82	83	19	82	96
72	16	83	64	01	72	37	84	19	28	53	63	26	28	95	82	61	24	73	64
15	34	24	25	37	48	59	60	98	92	83	74	76	34	25	46	57	39	62	18
75	37	38	29	30	17	64	72	34	65	37	62	16	35	48	58	91	63	74	94
65	27	87	02	31	86	26	28	35	85	92	16	85	46	72	82	93	74	62	26
11	75	62	93	53	75	82	16	62	42	36	91	74	23	25	36	94	64	18	92
92	75	32	74	37	29	85	67	82	34	76	82	37	46	29	85	74	62	91	83

13	24	35	46	57	68	79	80	91	34	26	37	94	57	35	24	37	58	69	21
34	25	13	48	96	87	76	65	54	45	23	13	86	57	46	35	21	90	86	14
36	27	38	49	50	97	86	75	64	56	67	78	89	09	17	98	07	53	24	36
23	34	45	56	78	98	91	41	90	54	36	14	10	39	48	56	23	34	45	56
13	45	97	13	65	75	87	18	07	68	12	35	46	75	98	78	67	56	34	23
93	89	17	43	23	34	45	56	67	89	90	01	13	40	41	48	33	44	22	13
23	23	14	36	47	58	92	35	46	58	69	70	98	75	64	23	10	29	88	54
99	38	29	07	67	46	53	42	13	54	65	76	87	98	09	10	89	23	45	65
76	56	87	98	10	34	09	87	64	12	34	67	89	56	47	65	34	25	46	87
89	10	86	75	01	34	56	16	72	35	46	76	01	98	70	54	32	43	56	78
56	45	34	23	98	10	98	08	97	86	75	46	79	11	12	38	49	10	80	14
45	45	12	35	46	57	68	90	10	78	16	75	82	30	49	58	01	45	67	89
34	56	46	23	34	56	67	78	89	09	18	50	10	11	34	56	75	23	45	65

89	30	29	30	49	58	67	58	49	38	27	11	40	59	78	26	38	56	20	38
15	49	30	78	21	30	78	21	30	41	67	58	49	37	25	36	47	58	69	70
18	48	57	68	79	80	21	36	45	36	36	58	67	59	36	25	37	46	58	69
22	35	46	57	68	79	80	29	38	47	38	29	56	47	38	29	56	47	38	41
14	83	49	57	68	79	57	46	35	25	36	47	58	69	71	59	60	71	89	22
44	17	69	78	69	58	47	36	47	58	69	70	89	68	57	33	41	89	58	59
99	04	58	69	03	92	83	84	75	68	79	80	68	57	47	36	47	58	69	70
12	39	49	58	67	59	40	29	51	60	48	59	38	27	36	58	49	58	67	23
17	69	58	47	27	48	59	68	77	36	49	50	28	39	47	68	58	11	92	81
45	36	47	58	69	70	89	37	26	35	47	58	69	70	89	13	28	38	47	56
15	48	59	60	79	26	37	48	55	67	58	49	20	39	48	67	58	41	20	19
15	93	04	92	83	74	57	68	79	80	29	38	47	56	37	28	39	40	59	68
34	86	74	63	25	36	47	58	69	70	81	92	34	56	55	36	54	36	47	58
21	30	41	50	61	79	57	68	79	68	57	46	35	24	35	43	57	68	79	81
13	45	67	78	69	70	89	67	56	45	36	47	58	69	70	89	27	38	49	50
11	45	67	78	89	90	98	76	76	54	45	65	34	23	45	43	25	36	47	58
10	12	03	49	58	67	58	67	58	49	57	68	59	40	39	28	37	46	57	66
44	30	49	58	67	58	49	30	29	30	29	30	49	58	67	59	40	21	31	17

FIG. 1. An Example of the 8-, 13-, and 18-digit Addition Problems.

Drill materials.—The drill materials consisted of:

(1) The 100 addition combinations formed from the digits 0 to 9, including zero and reverse digit combinations such as, 0 + 0, 0 + 1, 1 + 0, 8 + 9, 9 + 8, and 9 + 9.

(2) The 400 combinations formed from the figures 0 to 49. The 100 formed from the digits 0 to 9 were not included in these 400 combinations. The combinations (0 + 49, 49 + 0, 9 + 49, 49 + 9) were formed by adding one-place numbers to two-place numbers.

(3) The 500 combinations (0 + 99, 99 + 0, 9 + 99, 99 + 9) formed from the numbers 0 to 99 and constructed in the same manner as were the 400 combinations. The 100 combinations formed from the digits 0 to 9 and the 400 from the numbers 0 to 49 were not included in these 500.

Test materials.—The test materials consisted of four tests of equal difficulty. Each test contained 20 problems in each of three

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
43967	20827	93704	34760	37920	34760	35542	43907	93214	39075
26269	12729	25475	15275	19752	15275	18761	27934	61736	23496
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
74853	32527	39504	97162	58322	37521	33551	64012	79584	20897
47356	19489	27657	83470	24298	14690	22576	57361	59947	14698
(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
46826	86013	68964	85910	37853	41637	28507	32582	60675	45708
27358	54235	49487	26827	19476	24965	18664	15647	31968	39547
(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)
52138	44196	74612	37584	50983	48237	20018	87321	62091	91858
49248	32397	53956	17692	21695	29345	15384	53354	53485	90979
(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)
21923	30644	91662	62264	24194	66282	73945	89345	51092	62315
19475	19267	85943	41576	15269	47947	54696	30456	15269	46544

FIG. 2. An Example of the Subtraction Problems.

lengths of addition problems (8-, 13-, and 18-digit), 50 five-place problems in subtraction, 16 three-place problems in multiplication, and 12 division problems in which five-place numbers were to be divided by two-place numbers. Examples of each of these types of problems are given in Figures 1 to 4.

The addition problems of the three different lengths were used in order to determine whether a knowledge of the higher number combinations would produce greater changes in efficiency in long problems than it would in short ones. In the short (8-digit) problems there was little need for knowledge of the higher combinations, but, in the long problems (18-digit), computation involved combinations including

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
867	789	763	976	376	678	839	974
374	398	948	843	849	746	476	863

(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
473	847	938	683	684	734	864	863
968	693	746	479	738	986	938	497

FIG. 3. An Example of the Multiplication Problems.

the largest of those used for drill. The adding of the 13-digit problems also involved the higher number combinations but to a lesser extent than those problems 18 digits high.

Procedure.—The problem-solving ability of each subject was measured at four different times. The tests were given at intervals of seven days, during which time the subject was drilled on number facts in addition. All six types of problems in each test were given at one sitting—those in the first measurement of efficiency being designated as Test 1, those in the second as Test 2, those in the third as Test 3, and those in the fourth as Test 4.

Five minutes were allowed for the execution of each set of problems and a five-minute rest period was permitted between tests. The sequence of types of problems was determined by writing the names of the different types on six slips of paper, shuffling the slips of paper, and giving the problems in the order in which they were drawn. No preliminary drill work or training of any kind was given preceding Test 1.

Following Test 1 each subject was given a sheet containing the 100 combinations formed from the digits 0 to 9 and was asked to

(1)	(2)	(3)	(4)
39) 87538	48) 34871	81) 35429	79) 78251
(5)	(6)	(7)	(8)
69) 59763	97) 17496	93) 31735	46) 98764
(9)	(10)	(11)	(12)
36) 25684	73) 63824	67) 72439	87) 27948

FIG. 4. An Example of the Division Problems.

study these combinations at home until they were thoroughly known. Following this period of study, each subject was drilled on the 100 combinations until he had mastered them, and then he was given Test 2. Following Test 2, each subject was given a similar drill on the 400 combinations, 0 to 49, and after Test 3 he was drilled in this manner on the 500 combinations, 0 to 99, before being given the final test.

Scoring.—The addition problems were scored on the basis of the number of columns attempted and the number of columns correctly added. The subtraction, multiplication, and division problems were scored on the basis of the number of operations attempted and the number of correct operations.

CHANGES IN ADDING EFFICIENCY

8-digit problems.—The number of columns attempted and the number correctly added in the 8-digit problems for the four tests are given in Table I. On Test 1, which was the initial test preceding drill on the number facts of addition, the range in number of columns attempted in the 8-digit addition problems was 14 to 40, and in number of columns correctly added, 10 to 40. There were 20 two-column problems included in the test, so 40 columns were the maximum number that could be attempted or correctly solved. Of the 23 subjects, three reached the maximum in number of columns attempted, but only one added the entire 40 columns correctly. An examination of the individual scores

TABLE I. NUMBER OF COLUMNS ATTEMPTED AND NUMBER OF COLUMNS CORRECTLY ADDED IN 8-DIGIT ADDITION

Subject	Test 1		Test 2		Test 3		Test 4	
	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct
1	16	11	21	15	22	20	21	17
2	40	40	40	39	40	37	40	40
3	40	38	40	38	40	35	40	40
4	40	35	40	40	40	39	40	39
5	26	22	32	29	36	27	35	34
6	35	32	39	36	40	39	38	34
7	23	23	27	24	27	23	29	26
8	24	23	26	24	24	20	30	30
9	14	10	18	16	20	18	17	17
10	24	23	30	29	31	30	31	28
11	25	25	30	28	29	27	31	31
12	24	22	32	27	39	34	37	33
13	21	18	24	19	25	22	27	24
14	15	14	18	17	19	17	18	17
15	25	21	31	30	31	29	34	29
16	21	20	28	25	32	31	32	30
17	35	34	40	33	40	36	40	37
18	38	38	40	38	40	40	40	37
19	31	31	39	39	38	38	40	40
20	31	26	27	24	29	29	27	27
21	25	23	29	28	28	26	32	31
22	27	26	34	31	30	26	31	30
23	24	24	26	25	28	28	26	26
Average	27.1	25.2	30.9	28.4	31.7	29.2	32.0	30.3

made on Test 1 shows that for the majority of the subjects there was sufficient opportunity for the individuals participating in this experiment to make considerable improvement in their adding efficiency.

On Test 2, the range in the number of columns attempted was 18 to 40, and in the number of columns correctly solved it was 15 to 40. In comparison with Test 1 there was a slight reduction in the extent of the range of the number of columns attempted and the number of columns added correctly. The drill that was given before Test 3 was administered produced little change in the range of scores either for the number of columns attempted or for the number of columns correctly solved. Nor did significant changes in the range of scores follow the drill on the 500 combinations formed from the numbers 0 to 99, as is shown by the range of the number of columns attempted and the number of columns correctly added on Test 4.

Individual changes in number of columns attempted and number of columns correctly added in the 8-digit problems in Tests 2, 3, and 4, following drill in the number facts of addition are given in Table II. In comparing the number of columns attempted, one subject attempted 4 fewer columns on Test 2 than on Test 1, thus losing 12.90 per cent in efficiency; three subjects attempted exactly the same number of columns on Test 2 as they did on Test 1; and the remaining 19 subjects made an increase in the number of columns attempted, the increase ranging from 5.26 to 33.33 per cent. The average increase in number of columns attempted by the group on Test 2 over Test 1 is 13.96 per cent.

In comparing the number of columns correctly added on Test 2 with the number on Test 1, it is noted that there is a close relationship between the number of columns attempted and the number of columns correctly solved. On Test 2, three subjects added fewer columns correctly than they did on Test 1; two subjects added correctly the same number; and the remaining 18 subjects gained from 4.16 to 60.00 per cent in the number of columns correctly added. The group as a whole improved 12.95 per cent, which was only 1.01 per cent less than it gained in the number of columns attempted.

On Test 3 only one subject failed to attempt as many columns as he did on Test 1. This subject attempted two fewer columns, but increased by three the number of columns he solved correctly, thus making a percentage increase in efficiency of 11.54. Four subjects attempted the same number of columns on Test 3 as they did on Test 1, and the remaining 18 increased from 5.26 to 58.33 per cent. The group attempted 16.51 per cent more columns on Test 3 than on Test 1. In number of columns correctly added, three subjects were less efficient on Test 3 than on Test 1, one subject made no change, and 19 increased in efficiency. The average gain in number of columns correctly added on Test 3 over Test 1 was 15.89 per cent.

On Test 4 one subject failed to attempt as many columns as on Test 1, three made no change, and the remaining 19 increased the number of columns attempted by from 5.26 to 54.17 per cent. The average gain in number of columns attempted on Test 4 over Test 1 was 4.9

TABLE II. DIFFERENCES IN EFFICIENCY IN 8-DIGIT ADDITION RESULTING FROM DRILL ON THE NUMBER FACTS OF ADDITION

Subject	Difference in number attempted in Test 2 over Test 1		Difference in number correct in Test 2 over Test 1		Difference in number attempted in Test 3 over Test 1		Difference in number correct in Test 3 over Test 1		Difference in number attempted in Test 4 over Test 1		Difference in number correct in Test 4 over Test 1		Difference in number attempted in Test 3 over Test 2		Difference in number correct in Test 3 over Test 2		Difference in number attempted in Test 4 over Test 2		Difference in number correct in Test 4 over Test 2		Difference in number attempted in Test 4 over Test 3		Difference in number correct in Test 4 over Test 3	
	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent
1.	5	31.25	4	36.36	6	37.50	9	81.82	5	31.25	6	54.55	1	4.76	5	33.33	0	00.00	2	13.33	—1	—4.55	—3	—15.00
2.	0	00.00	—1	—2.50	0	00.00	—3	—7.50	0	00.00	0	00.00	0	00.00	—2	—5.13	0	00.00	1	2.56	0	00.00	3	8.11
3.	0	00.00	0	00.00	0	00.00	—3	—7.89	0	00.00	2	5.26	0	00.00	—3	—7.89	0	00.00	2	5.26	0	00.00	5	14.29
4.	0	00.00	0	00.00	0	00.00	4	11.43	0	00.00	4	11.43	0	00.00	—1	—2.50	0	00.00	2	5.50	0	00.00	0	00.00
5.	6	23.08	5	31.82	10	38.46	5	22.73	9	34.62	12	54.55	4	12.50	—2	—6.90	3	9.38	5	17.24	—1	—2.78	7	25.93
6.	4	11.42	4	12.50	5	14.29	7	21.88	3	8.57	2	6.25	1	2.56	3	8.33	—1	—2.56	—2	—5.56	—2	—5.00	—5	—12.82
7.	4	17.39	1	4.34	4	17.39	0	00.00	6	26.09	3	13.04	2	00.00	—1	—4.17	2	7.41	2	8.33	2	7.41	3	13.04
8.	0	00.00	0	00.00	0	00.00	—3	—13.04	6	25.00	7	30.43	—2	—7.69	—4	—16.67	4	15.38	6	25.00	—3	—15.00	10	50.00
9.	2	8.33	1	4.34	0	00.00	—3	—13.04	6	25.00	7	30.43	—2	—7.69	—4	—16.67	4	15.38	6	25.00	—3	—15.00	—1	—5.56
10.	4	26.67	2	8.33	1	4.34	0	00.00	3	21.43	7	70.00	—2	—11.11	—2	—12.50	—1	—3.33	—1	—3.45	0	00.00	—2	—6.67
11.	6	25.00	6	26.08	6	26.08	2	8.00	6	24.00	6	24.00	—1	—3.33	—1	—3.57	1	3.33	3	10.71	2	6.90	4	14.81
12.	8	33.33	3	12.00	4	16.00	2	8.00	6	24.00	6	24.00	—1	—3.33	—1	—3.57	1	3.33	3	10.71	2	6.90	4	14.81
13.	3	14.28	5	22.73	14	58.33	12	54.55	13	54.17	11	50.00	6	18.75	5	25.93	5	15.63	6	22.22	—1	—2.63	—1	—2.94
14.	3	14.28	1	5.56	4	19.05	4	22.24	6	26.59	6	33.33	1	4.17	3	15.79	5	12.50	5	26.32	2	8.00	2	9.09
15.	3	30.00	3	21.42	4	26.67	3	21.43	3	20.00	3	21.42	1	5.56	0	00.00	0	00.00	0	00.00	—1	—5.26	0	00.00
16.	6	24.00	9	42.85	6	24.00	8	38.10	8	36.00	8	38.10	0	00.00	—1	—3.33	3	9.68	3	9.68	—3	—9.68	0	00.00
17.	7	33.33	5	25.00	11	52.38	11	55.00	11	52.38	10	50.00	4	4.49	6	24.00	4	14.29	5	20.00	0	00.00	—1	—3.23
18.	5	14.28	—1	—2.94	5	14.28	5	5.88	5	14.28	3	8.82	0	00.00	3	9.68	0	00.00	4	12.12	0	00.00	—1	—2.75
19.	0	00.00	0	00.00	2	5.26	2	5.26	1	2.63	—1	—2.63	0	00.00	2	5.26	0	00.00	—1	—2.63	0	00.00	—3	—7.50
20.	8	25.81	7	22.58	7	22.58	9	29.03	9	29.03	1	3.85	—1	—2.56	—1	—2.56	1	2.56	1	2.56	—2	—5.26	—2	—5.26
21.	4	16.00	5	21.74	3	12.00	3	13.04	7	28.00	8	34.78	—1	—3.45	—2	—7.14	3	10.34	3	10.71	4	14.29	5	19.23
22.	7	25.92	5	19.23	3	11.11	0	00.00	4	14.81	4	15.38	—4	—11.76	—3	—16.13	—3	—8.82	—1	—3.23	1	3.33	4	15.38
23.	2	8.33	1	4.16	4	16.67	4	16.67	2	8.33	2	8.33	2	7.69	3	12.00	0	00.00	1	4.00	—2	—7.14	—2	—7.14
Average	3.8	13.96	3.3	12.95	4.5	16.51	4.0	15.89	4.9	17.96	5.1	20.38	0.7	2.25	0.7	2.60	1.1	3.52	1.9	6.58	0.4	1.24	1.1	3.87

columns, a percentage gain of 17.96. In comparing the number of columns correctly added on Test 4 with the number on Test 1, one subject solved fewer columns correctly, one solved the same number, and the other 21 subjects improved from 3.85 to 70.00 per cent. The group as a whole was 20.38 per cent more efficient on Test 4 than on Test 1.

The average difference in number of columns attempted on Test 3 over Test 2 was 0.7, a gain of only 2.25 per cent. The average difference in number of columns correctly added was also 0.7, with a percentage difference of 2.60. In comparing Test 4 with Test 2 the average difference in number of columns attempted was 1.1 and in number of columns correctly added it was 1.9, a percentage gain of 3.52 and 6.58 respectively. On Test 4 the average number of columns attempted was only 0.4 more than on Test 3, a gain of 1.24 per cent. There were 1.1 (3.87 per cent) more columns correctly added on Test 4 than on Test 3.

The average number of columns attempted and the average number of columns added correctly by the 23 subjects are given in Table III. An examination of the reliability of the differences between the average for Test 1 and the subsequent three tests shows that it is not statistically reliable. The highest reliability for both the number of columns attempted and the number of columns correctly added is found between Test 1 and Test 2. These reliabilities are high enough to be significant, but are not completely statistically reliable. The reliability of the differences between the averages for Test 2 and Test 3 and between those for Test 3 and Test 4 is only slightly higher than chance. The reliability of the differences between the number of columns attempted and the

TABLE III. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGES OF THE NUMBER OF COLUMNS ATTEMPTED AND THE NUMBER OF COLUMNS CORRECTLY ADDED IN 8-DIGIT ADDITION

	Test 1	Test 2	Test 3	Test 4	Test 1	Test 3	Test 2	Test 4
NUMBER ATTEMPTED—								
Average	27.1	30.9	31.7	32.0	27.1	31.7	30.9	32.0
S. D.	7.76	7.04	7.04	7.14	7.76	7.04	7.04	7.14
S. D. Average	1.62	1.46	1.46	1.48	1.62	1.46	1.46	1.48
Difference		3.8	0.8	0.3	4.9	4.6	0.8	1.1
S. D. Difference		2.18	2.06	2.08	2.19	2.18	2.06	2.08
Difference / S. D. Difference		1.74	0.39	0.14	2.23	2.11	0.39	0.53
NUMBER CORRECT—								
Average	25.2	28.4	29.2	30.3	25.2	29.2	28.4	30.3
S. D.	7.48	7.50	6.92	7.04	7.48	6.92	7.50	7.04
S. D. Average	1.56	1.56	1.44	1.46	1.56	1.44	1.56	1.46
Difference		3.2	0.8	1.1	5.1	4.0	0.8	1.9
S. D. Difference		2.20	2.12	2.05	2.14	2.12	2.12	2.14
Difference / S.D Difference		1.45	0.38	0.53	2.38	1.88	0.38	0.88

number of columns correctly added is slightly increased when the differences between Test 1 and Test 3 and between Test 1 and Test 4 are considered.

A more vivid picture of the effect of drill in the number facts of addition on efficiency in adding two-column problems 8 digits high may

be obtained by examining the curves in Figure 5. These four curves represent the number of 8-digit columns which each subject added correctly (Table I) on each of the four tests.

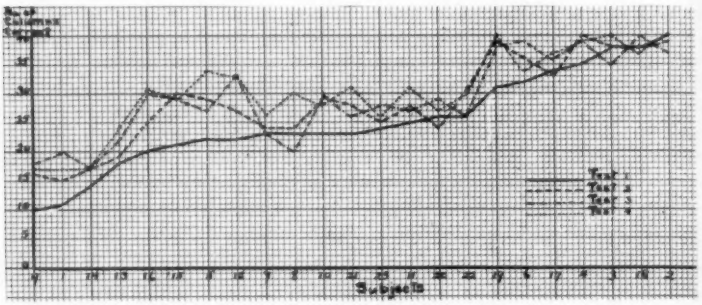

FIG. 5. Number of Columns Added Correctly in 8-digit Addition, Arranged According to Scores Made on Test 1.

Although the degree of efficiency, as shown by the number of columns attempted and the number of columns correctly added on the initial tests (Test 1), permitted ample opportunity for both individual and group improvement, drill in the number facts failed to produce any significant increase in efficiency in adding two-column problems 8 digits high. An examination of the curves in Figure 1 indicates clearly the lack of either significant individual improvement or great increase in group efficiency. Although these curves show a slight improvement resulting from drill in the number facts, this increase in efficiency is probably not much greater than should be expected from practice in column addition without the intervening drill on the number facts. Perhaps the most significant aspect of these curves is that the greatest increase in efficiency is between Test 1 and Test 2. The proximity of the curves for Tests 2, 3, and 4 shows the lack of improvement resulting from drill in the higher number facts of addition.

A consideration of all the data presented on the 8-digit problems indicates that the only improvement which might be considered as significant is shown on Test 2, which followed the drill on the 100 combinations formed from the digits 0 to 9. Drill on the higher number facts of addition failed to produce any significant improvement in efficiency.

13-digit problems.—The number of columns attempted and the number of colums added correctly by the 23 subjects in the 13-digit columns for the four tests are given in Table IV. On Test 1 the range in number of columns attempted is 9 to 38 and in number of columns correctly added it is 6 to 37. An examination of the scores shows that there was ample opportunity for improvement in efficiency in adding the 13-digit problems. An examination of the range of scores for Tests 2, 3, and 4 shows little variation from that of Test 1.

No significant changes in range of scores followed the drill on the number facts of addition.

Individual changes in number of columns attempted and number of columns correctly added in the 13-digit problems on Tests 2, 3, and 4 following drill on the number facts of addition are given in Table V. Both score differences and percentage differences are computed on the basis of the scores made on Test 1. Considering the group as a whole, there was some improvement in efficiency on Tests 2, 3, and 4. An examination of the data in Table V shows, however, that in many cases the differences between individual scores were negative rather than positive. On Test 2 only two of the 23 subjects attempted fewer columns than were attempted on Test 1, but seven subjects added fewer columns correctly. On Test 3, three subjects attempted fewer columns than on Test 1, and 10 were less efficient in the number of columns correctly added. The scores on Test 4 showed a much higher positive difference when compared with Test 1 than did those of Test 3. On Test 4 only one of the 23 subjects attempted fewer columns than on Test 1 and only three added fewer columns correctly. Individual increases in efficiency were more marked on Test 4 than on Test 2. In comparing the scores made on Test 3 with those on Test 2, eight of the 23 subjects attempted fewer columns on Test 3 and 10 added fewer columns correctly. Only one subject attempted fewer columns on Test 4 than on Test 2, but five added fewer columns correctly. Four subjects attempted fewer columns on Test 4 than they had on Test 3 and seven were less efficient as measured by the number of columns correctly added.

TABLE IV. NUMBER OF COLUMNS ATTEMPTED AND NUMBER OF COLUMNS CORRECTLY ADDED IN 13-DIGIT ADDITION

Subject	Test 1		Test 2		Test 3		Test 4	
	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct
1	12	10	12	6	12	8	13	12
2	38	37	40	37	40	36	40	36
3	23	19	24	20	26	20	25	22
4	22	18	24	22	24	21	25	18
5	16	9	20	12	19	13	20	17
6	20	18	22	16	19	15	22	21
7	15	14	15	11	16	13	16	13
8	15	11	13	9	12	10	15	12
9	10	6	10	7	10	6	10	9
10	15	14	17	17	19	18	19	17
11	16	13	19	16	19	13	19	16
12	15	13	15	13	18	15	15	13
13	14	12	14	12	14	8	15	14
14	9	9	10	10	9	8	12	11
15	13	10	13	9	15	9	20	17
16	14	12	17	13	16	13	19	15
17	19	17	20	16	22	18	20	14
18	20	19	23	22	24	24	24	23
19	18	14	22	19	21	17	22	16
20	18	12	16	14	16	14	15	12
21	13	13	15	11	14	12	16	13
22	15	14	20	17	18	13	20	18
23	11	10	13	13	13	12	15	14
Average	16.6	14.1	18.0	14.9	18.1	14.6	19.0	16.2

TABLE V. DIFFERENCES IN EFFICIENCY IN 13-DIGIT ADDITION RESULTING FROM DRILL ON THE NUMBER FACTS OF ADDITION

Subject	Difference in number attempted in Test 2 over Test 1		Difference in number correct in Test 2 over Test 1		Difference in number attempted in Test 3 over Test 1		Difference in number correct in Test 3 over Test 1		Difference in number attempted in Test 4 over Test 1		Difference in number correct in Test 4 over Test 1		Difference in number attempted in Test 3 over Test 2		Difference in number correct in Test 3 over Test 2		Difference in number attempted in Test 4 over Test 2		Difference in number correct in Test 4 over Test 2		Difference in number attempted in Test 4 over Test 3		Difference in number correct in Test 4 over Test 3	
	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent
1	0	00.00	—4	—40.00	0	00.00	—2	—20.00	1	8.33	2	20.00	0	00.00	2	33.33	1	8.33	6	100.00	1	8.33	4	50.00
2	2	5.26	0	00.00	2	5.26	—1	—2.70	2	5.26	—1	—2.70	0	00.00	—1	—2.70	0	00.00	—1	—2.70	0	00.00	0	00.00
3	1	4.35	1	5.26	3	13.04	1	5.26	2	8.69	3	15.79	2	8.33	0	00.00	1	4.17	2	10.00	—1	—3.85	2	10.00
4	2	9.09	4	22.22	2	9.09	3	16.67	3	13.63	0	00.00	0	00.00	—1	—4.55	1	4.17	—4	—18.18	1	4.17	—3	—14.29
5	4	25.00	3	33.33	3	18.75	4	44.44	4	25.00	8	88.88	—1	—5.00	1	8.33	0	00.00	5	41.67	1	5.26	4	30.77
6	2	10.00	—2	—11.11	—1	—5.00	—3	—16.67	2	10.00	3	16.67	—3	—13.64	—1	—6.25	0	00.00	5	31.25	3	15.79	6	40.00
7	0	00.00	—3	—21.43	1	6.67	—1	—7.14	1	6.67	—1	—7.14	1	6.67	2	18.18	1	6.67	2	18.18	0	00.00	0	00.00
8	—2	—13.33	—2	—18.18	—3	—20.00	—1	—9.09	0	00.00	1	9.09	—1	—7.69	1	11.11	2	15.38	3	33.33	3	25.00	2	20.00
9	0	00.00	1	16.67	0	00.00	0	00.00	0	00.00	3	50.00	0	00.00	—1	—14.29	0	00.00	2	28.57	0	00.00	3	50.00
10	2	13.33	3	21.43	4	26.66	4	28.57	4	26.66	3	21.43	2	11.76	1	5.88	2	11.76	0	00.00	0	00.00	—1	—5.56
11	3	18.75	3	23.08	3	18.75	0	00.00	3	18.75	3	23.08	0	00.00	—3	—18.75	0	00.00	0	00.00	0	00.00	3	25.08
12	0	00.00	0	00.00	3	20.00	2	15.38	0	00.00	0	00.00	3	20.00	2	15.38	0	00.00	0	00.00	—3	—16.67	—2	—13.33
13	0	00.00	0	00.00	0	00.00	—4	—33.33	1	7.14	2	16.67	0	00.00	—4	—33.33	1	7.14	2	16.67	1	7.14	6	75.00
14	1	11.11	1	11.11	0	00.00	—1	—11.11	3	33.33	2	22.22	—1	—10.00	—2	—20.00	2	20.00	1	10.00	3	33.33	3	37.50
15	0	00.00	—1	—10.00	2	15.37	—1	—10.00	7	53.84	7	70.00	2	15.38	0	00.00	7	53.85	8	88.89	5	33.33	8	88.89
16	3	21.43	1	8.33	2	14.28	1	8.33	5	35.71	3	23.33	—1	—5.88	0	00.00	2	11.76	2	15.38	3	18.75	2	15.38
17	1	5.26	—1	—5.88	3	15.79	1	5.88	1	5.26	—3	—17.65	2	10.00	2	12.50	0	00.00	—2	—12.50	—2	—9.09	—4	—22.22
18	3	10.50	3	15.79	4	20.00	5	26.32	4	20.00	4	21.05	1	4.35	2	9.09	1	4.35	1	4.55	0	00.00	—1	—4.17
19	4	22.22	5	35.71	3	16.67	3	21.43	4	22.22	4	28.57	—1	—4.55	—2	—10.53	0	00.00	—3	—15.79	1	4.76	—1	—5.88
20	—2	—11.11	2	16.67	—2	—11.11	2	16.67	—3	—16.67	0	00.00	0	00.00	0	00.00	—1	—6.25	—2	—14.29	—1	—6.25	—2	—14.29
21	2	15.38	—2	—15.38	1	7.69	—1	—7.69	3	23.08	0	00.00	—1	—6.67	1	9.09	1	6.67	2	18.18	2	14.29	1	8.33
22	5	33.33	3	21.43	3	20.00	—1	—7.14	5	33.33	4	28.57	—2	—10.00	—4	—23.53	0	00.00	1	5.88	2	11.11	5	38.46
23	2	18.18	3	30.00	2	18.18	2	20.00	4	36.36	4	40.00	0	00.00	—1	—7.69	2	15.38	1	7.69	2	15.38	2	16.67
Average	1.4	8.66	0.8	5.55	1.5	9.18	0.5	3.70	2.4	14.70	2.1	12.04	0.1	0.48	—0.3	—1.75	1.0	5.56	1.3	8.80	1.0	5.29	1.6	11.01

The average gain in efficiency on Test 2 over Test 1 was 8.66 per cent in number of columns attempted and 5.55 per cent in number of columns correctly added. The average gain on Test 3 over Test 1 was 9.18 per cent in number of columns attempted and 3.70 per cent in number of columns correctly added. There was an average gain of 14.70 per cent in number of columns attempted on Test 4 over those on Test 1 and a 12.04 per cent gain in number of columns added correctly. A comparison of the average scores of Test 4 with those of Test 2 shows that there was a gain of 0.48 per cent in number of columns attempted and a loss of 1.75 per cent in number of columns correctly added. This indicates a loss of efficiency after drill on the combinations 0 to 49. A comparison of the average scores of Test 4 with those of Test 2 shows that the differences were positive; 5.56 per cent more columns were attempted and 8.80 per cent more solutions were correct. The drill on the combinations 0 to 99 was beneficial, as 5.29 per cent more columns were attempted on Test 4 than on Test 3 and 11.01 per cent more columns were correctly added.

The average number of columns attempted and the average number added correctly by the 23 pupils in the 13-digit columns are given in Table VI. An examination of this table shows that the differences between the average for Test 1 and that for each of the subsequent three tests are not statistically reliable. The highest reliability, found for the difference between the average number of columns correctly added on Test 1 and on Test 4, is high enough to be significant, but it is not completely statistically reliable. The reliabilities of the differences between the averages for Test 2 and Test 3 and between those for Test 3 and Test 4 are not much higher than chance.

The lack of any significant increases in efficiency as the result of drill on the number facts of addition may be seen by examining the

TABLE VI. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGES OF THE NUMBER OF COLUMNS ATTEMPTED AND THE NUMBER OF COLUMNS CORRECTLY ADDED IN 13-DIGIT ADDITION

	Test 1	Test 2	Test 3	Test 4	Test 1	Test 3	Test 2	Test 4
NUMBER ATTEMPTED—								
Average	16.6	18.0	18.0	19.0	16.6	18.0	18.0	19.0
S. D.	5.94	6.46	6.50	5.72	5.94	6.50	6.46	5.72
S. D. Average	1.24	1.35	1.35	1.19	1.24	1.35	1.35	1.19
Difference	1.40	0.00	1.00	2.40	1.40	0.00	1.00	
S. D. Difference	1.83	1.91	1.80	1.72	1.83	1.91	1.80	
Difference / S. D. Difference	0.77	0.00	0.55	1.16	0.77	0.00	0.55	
NUMBER CORRECT—								
Average	14.1	14.9	14.6	16.2	14.1	14.6	14.9	16.2
S. D.	5.92	6.42	6.44	5.54	5.92	6.44	6.42	5.54
S. D. Average	1.23	1.34	1.34	1.15	1.23	1.34	1.34	1.15
Difference	0.80	0.30	1.69	2.10	0.30	0.33	1.30	
S. D. Difference	1.81	1.89	1.77	1.68	1.81	1.90	1.77	
Difference / S. D. Difference	0.45	0.16	0.90	1.25	0.28	0.16	0.73	

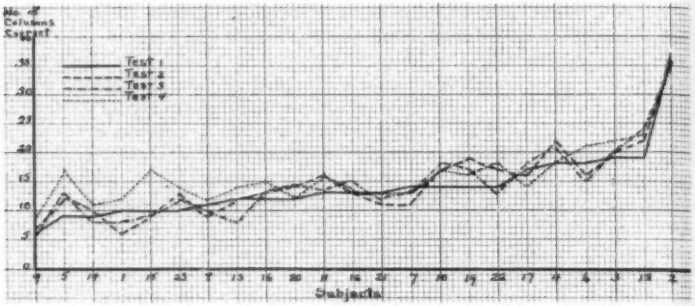

FIG. 6. Number of Columns Added Correctly in 13-digit Addition, Arranged According to Scores Made on Test 1.

curves in Figure 6. These curves represent the number of 13-digit columns each subject added correctly (Table IV) on each of the four tests. An examination of the curves in Figure 6 indicates clearly the lack of significant individual or group improvement resulting from drill on the number facts of addition.

A consideration of all the data presented concerning the performance of the 23 subjects on the 13-digit problems indicates that drill on the number facts of addition failed to produce in the longer problems as great a total increase in efficiency as the same drill produced in the short (8-digit) problems. The most marked increase in efficiency, however, followed the drill in the combinations 0 to 99. This fact provides some evidence that drill on the higher combinations produces a greater increase in efficiency in long problems than it does in short ones.

18-digit problems.—The number of columns attempted and the number of columns correctly added in the 18-digit problems of the four tests are given in Table VII. The range in number of columns attempted for Test 1 was 6 to 25; for Test 2, 6 to 27; for Test 3, 8 to 33; and for Test 4, 7 to 27. There was less relationship between the number of problems attempted and the number of problems correctly solved in the 18-digit problems than in the 8- or 13-digit problems, as shown by the ranges of scores for the number of problems correctly added. For Test 1 the range in scores for correctly solved columns was 1 to 23; for Test 2, 1 to 25; for Test 3, 3 to 28; and for Test 4, 4 to 24. Drill on the number facts of addition produced little change in the range of scores, the greatest difference being between Test 1 and Test 3.

Individual differences in number of columns attempted and number of columns correctly added on the four tests in the 18-digit problems are given in Table VIII. Both score differences and percentage differences are computed. A study of these data shows little individual gain in performance resulting from the drill on the combinations 0 to 9. In fact, seven of the 23 subjects attempted fewer columns on Test 2 than on Test 1, and eight added fewer columns correctly. The differences, however, between Test 1 and Test 3 are large; in all cases more columns

were attempted and only four subjects added fewer columns correctly. As a group, there was a gain of 35.24 per cent in number of columns attempted and an improvement in correct solutions of 28.57 per cent in Test 3 over Test 1. The group gain in number of columns attempted on Test 4 dropped to 18.01 per cent over Test 1, but there was an increase to 33.14 per cent in number of correct solutions.

There was a marked increase in efficiency on Test 3 over Test 2 (Table VIII). All 23 subjects attempted more columns, the group making an average improvement of 34.92 per cent. Three subjects solved fewer problems correctly, but for the group there was an average increase in correct solutions of 27.69 per cent. In comparing the scores made on Test 4 with those on Test 2, it is seen that 21 of the 23 subjects attempted more columns, with an average increase of 17.07 per cent. Six subjects added fewer columns correctly, but the loss for these subjects was slight and the remaining 17 subjects made an average group increase of 31.64 per cent. On Test 4, 18 of the 23 subjects attempted fewer columns than on Test 3. There was an average reduction of 12.72 per cent in number of columns attempted. Although nine subjects added fewer columns correctly on Test 4 than on Test 3, there was an average group gain of 3.56 per cent.

The average number of columns attempted and the average number of columns added correctly are given in Table IX. The highest reliability of the differences between the average of the number of columns attempted, comparing the tests in the order in which they

TABLE VII. NUMBER OF COLUMNS ATTEMPTED AND NUMBER OF COLUMNS CORRECTLY ADDED IN 18-DIGIT ADDITION

Subject	Test 1		Test 2		Test 3		Test 4	
	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct
1	9	5	9	5	12	6	8	4
2	25	23	27	25	33	28	27	24
3	14	8	13	10	17	10	16	12
4	13	13	8	7	20	16	17	12
5	11	5	12	2	14	7	14	13
6	12	5	12	11	18	13	16	9
7	10	9	10	10	13	3	10	8
8	7	1	6	1	10	6	10	7
9	6	4	8	2	9	8	7	6
10	10	5	10	7	14	10	14	14
11	12	10	11	4	14	7	12	11
12	10	9	9	7	12	8	11	10
13	9	5	9	6	11	3	9	7
14	7	6	6	5	8	6	7	6
15	6	2	9	7	13	8	11	6
16	9	5	9	7	12	8	12	11
17	13	9	14	11	18	14	14	11
18	13	12	14	11	16	14	15	14
19	12	10	10	7	16	11	14	13
20	9	7	10	9	13	7	10	8
21	9	7	9	7	12	12	10	8
22	9	7	12	8	14	11	15	10
23	9	8	9	8	11	9	9	9
Average	10.6	7.6	10.7	7.7	14.3	9.8	12.5	10.1

TABLE VIII. DIFFERENCES IN EFFICIENCY IN 18-DIGIT ADDITION RESULTING FROM DRILL ON THE NUMBER FACTS OF ADDITION

Subject	Difference in number attempted in Test 2 over Test 1		Difference in number correct in Test 2 over Test 1		Difference in number attempted in Test 3 over Test 1		Difference in number correct in Test 3 over Test 1		Difference in number attempted in Test 4 over Test 1		Difference in number correct in Test 4 over Test 1		Difference in number attempted in Test 3 over Test 2		Difference in number correct in Test 3 over Test 2		Difference in number attempted in Test 4 over Test 2		Difference in number correct in Test 4 over Test 2		Difference in number attempted in Test 4 over Test 3		Difference in number correct in Test 4 over Test 3	
	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent
1	0	00.00	0	00.00	3	33.33	1	20.00	−1	−11.11	−1	−20.00	3	33.33	1	20.00	−1	−11.11	−1	−20.00	−4	33.33	−2	−33.33
2	2	8.00	2	8.70	8	32.00	5	21.75	2	8.00	1	4.35	6	22.22	3	12.00	0	00.00	−1	−4.00	−6	−18.18	−4	−14.28
3	−1	−7.14	2	25.00	3	21.43	2	25.00	2	14.29	4	50.00	4	30.76	0	00.00	3	23.08	2	20.00	−1	−5.88	2	20.00
4	−5	−38.46	−6	−46.15	7	53.85	3	23.08	4	30.77	−1	−7.69	12	150.00	9	128.57	5	71.43	−3	−15.00	−4	−20.00	−4	−20.00
5	1	9.09	−3	−60.00	3	27.27	2	40.00	3	27.27	8	160.00	2	16.67	5	250.00	2	16.67	11	550.00	0	00.00	6	85.71
6	0	00.00	6	120.00	6	50.00	8	160.00	4	33.33	4	80.00	6	50.00	2	18.18	4	33.33	−2	−18.18	−2	−11.11	−4	−30.76
7	0	00.00	1	11.11	3	30.00	−6	−66.67	0	00.00	−1	−11.11	3	30.00	−7	−70.00	0	00.00	−2	−20.00	−3	−23.07	5	166.67
8	−1	−14.29	0	00.00	3	42.86	5	500.00	3	42.86	6	600.00	4	66.67	5	500.00	4	66.67	6	60.00	0	00.00	1	16.67
9	2	33.33	−2	−50.00	3	50.00	4	100.00	1	16.67	2	50.00	1	12.50	6	300.00	−1	−12.50	4	200.00	−2	−22.22	−2	−25.00
10	0	00.00	2	40.00	4	40.00	5	100.00	4	40.00	9	180.00	4	40.00	3	42.85	4	40.00	7	100.00	0	00.00	4	40.00
11	−1	−8.33	−6	−60.00	2	16.67	−3	−30.00	0	00.00	1	10.00	3	27.27	3	75.00	1	9.09	7	175.00	−2	−14.21	4	57.14
12	−1	−10.00	−2	−22.22	2	20.00	−1	−11.11	1	10.00	1	11.11	3	33.33	1	14.28	2	22.22	3	42.86	−1	−8.33	2	25.00
13	0	00.00	1	20.00	2	22.22	−2	−40.00	0	00.00	2	40.00	2	11.11	−3	−50.00	0	00.00	1	16.67	−2	−18.18	4	133.33
14	−1	−14.29	−1	−16.67	1	14.29	0	00.00	0	00.00	0	00.00	2	33.33	1	20.00	1	16.67	1	20.00	−1	−12.50	0	00.00
15	3	50.00	5	250.00	7	116.67	6	300.00	5	83.33	4	200.00	4	44.44	1	14.28	2	22.22	−1	−14.29	−2	−15.38	−2	−25.00
16	0	00.00	2	40.00	3	33.33	3	60.00	3	33.33	6	120.00	3	33.33	1	14.28	3	33.33	4	57.14	0	00.00	3	37.50
17	1	7.69	2	22.22	5	38.46	5	55.56	1	7.69	2	22.22	4	28.57	3	27.27	0	00.00	0	00.00	−4	−22.22	−3	−21.42
18	1	7.69	−1	−8.33	3	23.08	2	16.67	2	15.38	2	16.67	2	14.28	3	27.27	1	7.14	3	27.27	−1	−6.25	0	00.00
19	−2	−16.67	−3	−30.00	4	33.33	1	10.00	2	16.67	3	30.00	6	60.00	4	57.14	4	40.00	6	85.71	−2	−12.50	2	18.18
20	1	11.11	2	28.57	4	44.44	0	00.00	1	11.11	1	14.29	3	30.00	−2	−22.22	0	00.00	−1	−11.11	−3	−23.07	1	14.28
21	0	00.00	0	00.00	3	33.33	5	71.43	1	11.11	1	14.29	3	33.33	5	71.42	1	11.11	1	14.29	−2	−16.67	−4	−33.33
22	3	33.33	1	14.29	5	55.56	4	51.14	6	66.67	3	42.86	2	16.67	3	37.50	3	25.00	2	25.00	1	7.14	−1	−9.09
23	0	00.00	0	00.00	2	22.22	1	12.50	0	00.00	1	12.50	2	22.22	1	12.50	0	00.00	1	12.50	−2	−18.18	0	00.00
Average	0.1	0.82	0.1	1.14	3.7	35.24	2.2	28.57	1.9	18.01	2.5	33.14	3.7	34.92	2.1	27.69	1.8	17.07	2.4	31.64	−1.8	−12.72	0.3	3.56

were given, is between Test 2 and Test 3. This difference is so nearly a complete statistical reliability that one may conclude that the drill on the combinations formed from the digits 0 to 49 produced a significant increase in the number of columns attempted. The reliability of the differences between the averages of the number of columns correctly added on Test 2 and Test 3 is not as significant statistically as that between the averages of the number of columns attempted. The reliability of the difference between the average number of columns attempted and correctly solved on Test 1 and Test 2 is only slightly greater than chance, and the difference between those on Test 3 and Test 4 is not high enough to be significant. The difference between the averages of the number of columns attempted is higher than that between the averages of the number of columns correctly added. There is a slight increase in the reliability of the differences between the average number of columns attempted on Tests 1 and 3 and on Tests 1 and 4, but there is a decrease between Tests 2 and 4. For the number of columns correctly added, the reliability of the differences between the averages increases when Tests 1 and 3, 1 and 4, and 2 and 4 are compared.

The lack of any marked improvement in adding two-column problems, 18 digits high, resulting from drill on the number facts of addition may be seen by examining the curves in Figure 7. These curves represent the number of columns each subject added correctly on each of the four tests (Table VII). An examination of the curves in Figure 7 shows clearly the lack of any great individual or group improvement in efficiency resulting from drill on the number facts of addition. Perhaps the most significant aspect of these curves is that the greatest distance between lines, which indicates the greatest increase in efficiency, is between the curves for Tests 2 and 3. The proximity of the curves for Tests 1 and 2 and for Tests 3 and 4 shows the lack of improvement on these tests.

TABLE IX. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGES OF THE NUMBER OF COLUMNS ATTEMPTED AND THE NUMBER OF COLUMNS CORRECTLY ADDED IN 18-DIGIT ADDITION

	Test 1	Test 2	Test 3	Test 4	Test 1	Test 3	Test 2	Test 4
NUMBER ATTEMPTED—								
Average	10.6	10.7	14.3	12.5	10.6	14.3	10.7	12.5
S. D.	3.68	4.34	4.78	4.22	3.68	4.78	4.34	4.22
S. D. Average	0.76	0.91	0.99	0.89	0.76	0.99	0.91	0.89
Difference		0.1	3.6	1.8	1.9	3.7	3.6	1.8
S. D. Difference		1.19	1.35	1.33	1.17	1.25	1.35	1.29
Difference / S. D. Difference		0.09	2.66	1.35	1.62	2.96	2.66	1.39
NUMBER CORRECT—								
Average	7.6	7.7	9.8	10.1	7.6	9.8	7.7	10.1
S. D.	4.36	4.52	5.26	4.06	4.36	5.26	4.52	4.06
S. D. Average	0.91	0.94	1.09	0.84	0.91	1.09	0.94	0.84
Difference		0.1	2.1	0.3	2.5	2.2	2.1	2.4
S. D. Difference		1.31	1.44	1.38	1.24	1.42	1.44	1.26
Difference / S. D. Difference		0.08	1.46	0.22	2.01	1.55	1.46	1.90

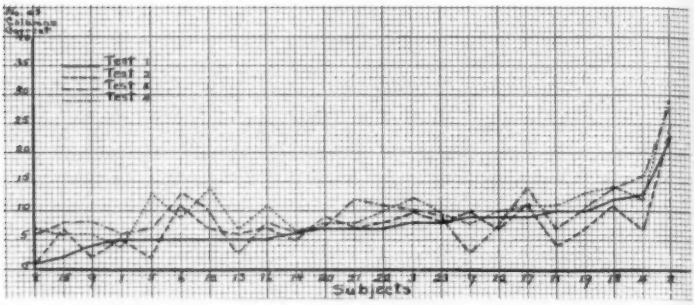

FIG. 7. Number of Columns Added Correctly in 18-digit Addition, Arranged According to Scores Made on Test 1.

A consideration of all the data presented concerning the performance on the 18-digit problems indicates that the only improvement which might be considered as significant is shown on Test 3, which followed drill on the combinations formed from the number 0 to 49. Drill on the combinations 0 to 9 produced only a slight increase in efficiency and, although there was an improvement in adding following drill on the combinations 0 to 99, it was less marked than that on Test 3. These data present little evidence that drill on the higher combinations (0 to 99) will produce significant increases in efficiency in long problems (18-digit).

8-, 13-, and 18-digit problems.—A comparison of changes in the number of columns added correctly in the 8-, 13-, and 18-digit problems following drill on the number facts of addition provides some interesting as well as confusing data. The percentage increases in efficiency in the number of columns correctly added on Tests 2, 3, and 4 over Test 1 for each of the three length problems are plotted in the curves in Figure 8.

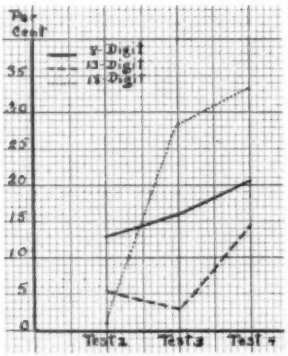

FIG. 8. Comparison of the Percentage Increases in Efficiency in Addition on the 8-, 13-, and 18-digit Problems.

In relation to some of the proposals concerning the teaching of the higher combinations, it might be expected that the least increase in efficiency would be found in the 8-digit problems and the greatest in the 18-digit problems. The results of this study show (Figure 8) that the least change in efficiency was in the 13-digit problems and the greatest in the 18-digit.

An examination of the curves in this figure shows definitely that for the 8-digit problems, drill on the combinations 0 to 9 was the most beneficial and drill on the higher combinations produced relatively little increase in efficiency.

For the 13-digit problems, drill on either the combinations 0 to 9 or 0 to 49 produced little significant increase in efficiency, but drill on the combinations 0 to 99 produced what might be considered a significant increase in adding efficiency.

In the 18-digit problems, drill on either the 0 to 9 or the 0 to 99 combinations produced little increase in efficiency, but drill on the 0 to 49 combinations produced a significant increase. Although these data do not entirely substantiate the claim that drill on the higher combinations will produce significant increases in efficiency in adding long problems, there is some evidence that drill on the higher combinations is more valuable than that on the 100 combinations formed from the digits 0 to 9.

TRANSFER VALUE OF DRILL IN THE NUMBER FACTS OF ADDITION

Another aspect of this study was the measurement of changes in efficiency in computing problems in subtraction, multiplication, and division following drill in the number facts of addition.

Subtraction.—The number of subtraction operations attempted and the number of correct operations for each subject for the four tests are given in Table X. Since each problem was scored on the basis of the number of operations, and each problem contained five operations, the highest possible score in this division was 250. The range in number of operations attempted was: Test 1, 68 to 240; Test 2, 70 to 250; Test 3, 85 to 250; and Test 4, 68 to 250. The range for correct operations was: Test 1, 67 to 232; Test 2, 65 to 240; Test 3, 84 to 247; and Test 4, 66 to 248. There is a considerable increase in the range of number of operations attempted on Tests 2 and 3 as compared with Test 1, but on Test 4 the range was not greatly different from that of Test 1. There was a decrease in the range of correct operations on Test 2 as compared with Test 1, but on Test 3 there was an increase. On Test 4 the range of correct operations was similar to that on Test 1.

Individual changes in the number of operations attempted and the number of operations correctly executed are given in Table XI. Both score differences and percentage differences for Tests 2, 3, and 4 are computed on the basis of the scores made on Test 1. In comparing the scores of Test 2 with those of Test 1, only one subject attempted fewer operations and only one subject (the same one) solved fewer operations

TABLE X. NUMBER OF OPERATIONS ATTEMPTED AND NUMBER OF OPERATIONS COMPLETED CORRECTLY IN SUBTRACTION

Subject	Test 1		Test 2		Test 3		Test 4	
	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct
1	68	67	70	65	85	84	68	66
2	207	202	223	220	145	138	250	246
3	216	214	220	217	250	247	250	248
4	240	232	250	240	250	246	250	241
5	152	150	201	193	185	182	198	190
6	148	147	195	186	188	187	218	215
7	109	107	127	126	124	132	125	124
8	130	124	159	153	155	153	165	155
9	100	98	176	126	148	141	147	129
10	151	149	195	194	175	174	200	197
11	106	103	125	124	123	116	146	142
12	173	169	195	191	204	200	226	221
13	125	124	145	142	151	151	162	159
14	98	96	127	125	135	132	140	135
15	173	172	225	223	225	219	250	243
16	145	134	176	174	180	174	201	198
17	192	183	139	123	228	218	250	235
18	145	141	145	145	160	158	161	160
19	193	192	208	208	223	221	249	246
20	150	144	178	172	192	187	215	210
21	144	140	150	148	148	141	170	169
22	140	138	153	151	155	154	186	185
23	135	133	152	151	155	153	169	167
Average	149.6	146.0	169.3	165.1	173.7	169.9	191.1	186.6

correctly. For the group as a whole there were 13.23 per cent more operations attempted on Test 2 than were attempted on Test 1 and there were 13.13 per cent more correct solutions. This indicates that drill on the number facts 0 to 9 produced an increase in efficiency in solving problems in subtraction. A comparison of the scores on Test 3 with those on Test 1 shows that only one subject attempted and solved correctly fewer operations than on Test 1. There was an average group gain of 15.53 per cent in number of operations attempted and 16.67 per cent in correct operations. On Test 4, all 23 subjects attempted more operations than were attempted on Test 1, and there was an average improvement of 27.79 per cent. One subject executed one less operation correctly on Test 4 than on Test 1, but the group as a whole was 27.74 per cent more efficient on Test 4 than on Test 1.

A comparison of Test 3 with Test 2 shows that seven subjects attempted fewer operations and six were less efficient in number of correct operations. However, there was an average group improvement of 2.54 per cent in number of operations attempted, and of 2.92 per cent in number of correct operations. The small improvement on Test 3 as compared with Test 2 shows that the drill on the combinations 0 to 49 was not as effective as was the drill on the combinations 0 to 9.

Drill on the combinations 0 to 99 produced almost as significant an increase in efficiency as did drill on the combinations 0 to 49, as is shown by a comparison of the scores made on Test 4 with those made on Test 3. There was an average increase of 10.07 per cent in number

TABLE XI. DIFFERENCES IN EFFICIENCY IN SUBTRACTION RESULTING FROM DRILL ON THE NUMBER FACTS OF ADDITION

Subject	Difference in number attempted in Test 2 over Test 1		Difference in number correct in Test 2 over Test 1		Difference in number attempted in Test 3 over Test 1		Difference in number correct in Test 3 over Test 1		Difference in number attempted in Test 4 over Test 1		Difference in number correct in Test 4 over Test 1		Difference in number attempted in Test 3 over Test 2		Difference in number attempted in Test 4 over Test 2		Difference in number correct in Test 3 over Test 2		Difference in number correct in Test 4 over Test 2		Difference in number attempted in Test 4 over Test 3		Difference in number correct in Test 4 over Test 3	
	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent
1	2	2 94	2	2 99	17	25 00	17	25 35	0	00 00	—1	—1 49	15	21 43	19	29 23	—2	—2 86	1	1 54	—17	—20 00	—18	—21 43
2	16	7 73	18	8 91	—62	—29 95	—64	—31 69	43	20 77	44	21 78	—76	—34 96	—82	—37 27	27	12 11	26	11 82	105	72 41	106	78 26
3	4	1 85	3	1 40	34	15 74	33	15 42	34	15 74	34	15 84	30	13 64	30	13 82	30	13 64	31	14 29	0	00 00	1	0 40
4	10	4 17	8	3 45	10	4 17	14	6 46	10	4 17	9	3 88	0	00 00	6	2 50	0	00 00	1	0 42	0	00 00	—5	—2 03
5	49	32 22	43	28 67	33	21 71	32	21 33	46	30 26	40	26 67	—16	—7 96	—11	—5 70	—3	—1 49	—3	—1 55	13	7 03	8	4 40
6	47	31 76	39	26 53	40	27 03	40	27 28	70	47 30	68	46 26	—7	—3 59	1	0 54	23	11 79	29	15 50	30	15 96	28	14 97
7	18	16 15	19	17 76	25	22 94	25	23 36	16	14 68	17	15 89	—7	—5 51	6	4 76	—2	—1 57	—2	—1 59	—9	—6 72	—8	—6 06
8	29	22 31	29	23 39	25	19 23	29	23 39	35	26 92	31	25 00	—4	—2 32	0	00 00	6	3 77	2	1 31	10	6 45	—2	1 31
9	36	36 00	28	28 57	48	48 00	43	43 88	47	47 00	41	41 84	12	8 82	15	11 90	11	8 09	13	10 32	1	0 68	—2	1 42
10	44	29 14	45	30 20	24	15 89	25	16 67	49	32 45	48	32 21	—20	—10 26	—20	—10 31	5	2 56	3	1 55	25	14 29	23	13 22
11	19	17 92	21	20 39	17	16 04	13	12 62	40	37 74	39	37 86	—2	—1 60	—8	—6 45	21	16 80	18	14 52	23	18 70	26	22 41
12	22	12 72	21	12 43	31	17 98	31	18 34	53	30 64	52	30 77	9	4 62	9	4 71	31	15 90	30	15 71	22	10 78	21	10 50
13	20	16 00	18	14 52	26	20 80	27	21 77	37	29 60	35	28 23	6	4 14	7	6 34	17	11 72	17	11 97	11	7 28	8	8 00
14	29	29 59	29	30 21	37	37 76	36	37 57	42	42 86	39	40 63	8	6 30	7	5 60	13	10 24	10	8 00	5	3 70	3	2 27
15	52	30 06	51	29 65	52	30 06	47	27 32	77	44 51	71	41 28	0	00 00	—4	—1 79	25	11 11	20	8 97	25	11 11	24	10 96
16	22	22 07	40	29 85	35	24 14	40	29 85	56	38 62	64	47 76	3	1 69	0	00 00	24	13 56	24	13 79	21	11 67	24	13 79
17	—53	—27 60	—60	—32 24	36	18 75	35	19 13	58	30 21	52	28 31	89	64 03	95	77 24	111	79 86	112	80 166	22	0 65	17	7 80
18	0	00 00	4	2 84	18	10 34	17	12 05	16	11 03	19	13 48	15	10 34	13	8 97	16	11 03	15	10 34	2	1 38	2	1 97
19	15	7 77	16	8 33	30	15 54	29	15 11	56	29 02	54	28 13	15	7 21	13	6 25	41	19 71	38	18 27	26	11 66	25	11 31
20	28	18 67	28	19 44	42	28 00	43	22 91	65	43 33	66	45 83	14	7 87	15	8 72	37	23 79	38	22 09	23	11 98	23	12 30
21	6	4 17	8	5 71	4	2 89	1	0 71	26	18 06	29	20 71	—2	—1 33	—7	—4 73	20	13 33	21	14 19	22	14 86	28	19 86
22	13	9 29	13	9 42	15	10 71	16	11 59	46	32 86	47	34 06	2	1 31	3	1 99	33	21 57	34	22 52	31	21 00	31	20 13
23	17	11 85	18	13 53	20	14 81	20	15 04	34	25 19	34	25 56	3	1 97	2	1 32	17	11 18	16	10 60	14	9 03	14	9 15
Average	19 8	13 23	19 2	13 13	24 1	15 53	23 9	16 67	41 6	27 79	40 5	27 74	4 3	2 54	4 8	2 92	21 8	12 86	21 3	13 01	17 4	10 07	16 5	9 80

TABLE XII. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGES OF THE NUMBER OF OPERATIONS ATTEMPTED AND THE NUMBER OF OPERATIONS COMPLETED CORRECTLY IN SUBTRACTION

	Test 1	Test 2	Test 3	Test 4	Test 1	Test 3	Test 2	Test 4
NUMBER ATTEMPTED—								
Average	149.6	169.3	173.7	191.1	149.6	173.7	169.1	191.1
S. D.	40.50	42.45	41.70	49.15	40.50	41.70	42.45	49.15
S. D. Average	8.44	8.84	8.70	10.24	8.44	8.70	8.84	10.24
Difference		19.7	4.4	17.4	41.5	24.1	4.4	21.8
S. D. Difference		12.22	12.41	13.44	13.26	12.11	12.41	13.54
Difference / S. D. Difference		1.61	0.35	1.29	3.13	1.98	0.35	1.61
NUMBER CORRECT—								
Average	146.0	165.1	169.9	186.6	146.0	169.9	165.1	186.6
S. D.	37.74	40.95	38.10	47.10	37.74	38.10	40.95	47.10
S. D. Average	7.86	8.52	7.94	9.81	7.86	7.94	8.52	9.81
Difference		19.1	4.8	16.7	40.6	23.9	4.8	21.5
S. D. Difference		11.58	11.64	12.62	12.56	11.17	11.64	13.00
Difference / S. D. Difference		1.65	0.41	1.32	3.23	2.14	0.41	1.58

of operations attempted and a 9.80 per cent increase in number of operations correctly solved.

The average number of operations attempted and the average number of operations correctly executed are given in Table XII. An examination of the reliability between the averages for the different tests shows that the difference (both in attempts and in corrects) between the averages of Test 1 and Test 4 is statistically reliable. This fact indicates that there was a definite and significant transfer value from drill on the number facts of addition to the working of problems in subtraction. The highest statistical reliability in the differences between tests, taken in the order in which they were given, is the difference between the averages for Test 1 and Test 2, and for Test 3 and Test 4. These differences, although not entirely statistically reliable, are high enough to be significant and indicate that the greatest transfer value came from drill on the combinations 0 to 9 and 0 to 99. For some reason which the data do not show, drill on the combinations 0 to 49 produced but little increase in efficiency in working problems in subtraction.

A more adequate picture of the transfer value of drill on the number facts of addition to the working of subtraction problems may be had by examining the curves in Figure 9. These four curves represent the number of operations each subject solved correctly (Table X) on each of the four tests. The greatest distance between curves, which shows the greatest increase in efficiency, is between the curves for Tests 1 and 2, and for Tests 3 and 4. The distance between the curves for Tests 1 and 2 is greater than the distance between the curves for Tests 3 and 4. The proximity of curves for Tests 2 and 3 indicate little increase in efficiency in Test 3 over Test 2.

A consideration of all the data presented concerning the transfer value of drill in the addition number facts to problem computation in subtraction shows that there is a significant transfer value; that is, drill in the number facts of addition will increase efficiency in subtrac-

tion. Further, these data show that the most significant transfer value was derived from drill on the combinations 0 to 9; that there was little transfer value from drill on the combinations 0 to 49; but that drill on the combinations 0 to 99 produced an additional increase in efficiency over that resulting from drill on the 0 to 9 and the 0 to 49 combinations.

Multiplication.—The number of operations attempted and the number of operations correctly executed in the multiplication problems by the 23 subjects on each of the four tests are given in Table XIII. Each problem was scored on the basis of its four operations, three in multiplication and one in addition. Since there were 16 problems, the maximum score for any test is 64. The range in number of operations attempted for Test 1 was 14 to 64 and for the number of correct operations 11 to 55. There was a marked reduction in the range of scores for Test 2, following drill on the addition combinations formed from the digits 0 to 9. The range in number of operations attempted in Test 2 was 24 to 64 and for the number of correct operations it was 23 to 62. There was little change in the range of number of operations attempted on Test 3, 22 to 64, as compared to that on Test 2, but the range of number of correct operations, 13 to 62, increased. The range in number of operations attempted for Test 4 was the same as for Test 3, but there was a slight reduction in the range of scores, 19 to 62, for the number of correct operations.

Individual changes in number of operations attempted and number of operations correctly executed in the multiplication problems on Tests 2, 3, and 4 are given in Table XIV. Both score differences and percentage differences are computed on the basis of Test 1. A comparison of

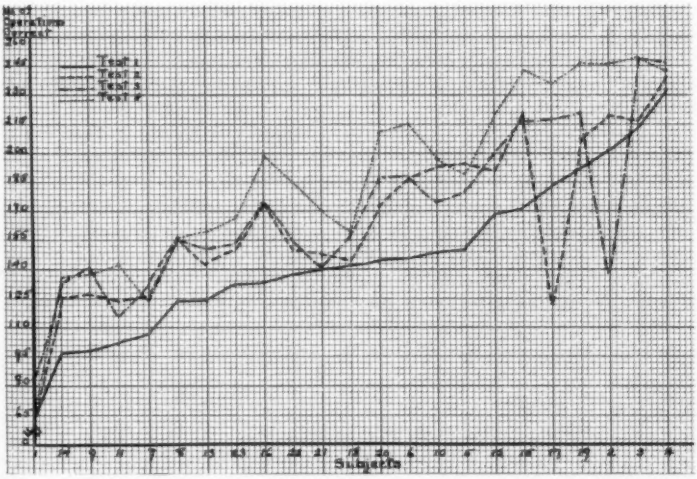

FIG. 9. Number of Subtraction Problem Operations Completed Correctly, Arranged According to the Scores Made on Test 1.

TABLE XIII. NUMBER OF OPERATIONS ATTEMPTED AND NUMBER OF OPERATIONS COMPLETED CORRECTLY IN MULTIPLICATION

Subject	Test 1		Test 2		Test 3		Test 4	
	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct
1	21	16	27	23	44	41	31	28
2	58	55	64	54	64	62	64	62
3	52	45	64	62	64	61	64	61
4	64	53	64	62	64	62	64	62
5	28	24	44	36	40	34	42	39
6	40	34	52	47	56	49	53	50
7	29	27	38	38	35	29	36	34
8	24	16	36	31	34	30	39	35
9	16	11	28	23	26	16	22	19
10	34	31	45	41	47	44	48	43
11	14	13	24	23	22	13	24	22
12	31	30	48	46	40	38	45	43
13	27	27	38	32	36	33	39	35
14	19	17	26	26	27	26	25	24
15	35	29	51	45	46	43	44	37
16	23	14	36	29	33	27	40	39
17	40	31	61	52	59	47	56	50
18	36	36	40	40	43	42	43	43
19	50	48	53	51	59	57	60	55
20	41	39	59	56	54	54	56	52
21	28	26	32	30	31	29	32	31
22	26	24	39	37	34	31	40	39
23	20	18	34	33	33	30	40	40
Average	32.9	28.9	43.6	39.9	43.1	39.0	43.8	41.0

the number of operations attempted on Test 2 with those on Test 1 shows that all of the 23 subjects except one attempted more operations. The average gain for the group was 32.66 per cent. There was a greater gain (38.10 per cent) in number of operations correctly solved. The marked improvement on Test 2 indicates that there was a definite transfer value from drill on the 0 to 9 addition combinations to computing problems in multiplication.

There were 31.08 per cent more operations attempted on Test 3 than on Test 1 and 33.58 more correct solutions. A comparison of the average scores on Test 3 with those on Test 2 shows that the drill on the addition combinations 0 to 49 had a slightly negative transfer value. The average percentage gain, 33.20 per cent, in number of columns attempted on Test 4 was approximately the same as was shown on Test 1. The average increase of 42.02 per cent in number of correct operations indicates, however, that drill on the higher combinations had a positive transfer value.

On Test 3 there was an average reduction of 1.20 per cent in number of operations attempted and a 2.07 per cent reduction in number of correct operations. This is further evidence of the negative transfer value of drill on the 0 to 49 combinations. On Test 4 the group attempted only 0.40 per cent more operations than on Test 2 and the average gain in number of correct operations was only 2.83 per cent. On Test 4 there was an average gain over Test 3 of 1.61 per cent in operations attempted and 5.01 per cent gain in number of correct operations.

TABLE XIV. DIFFERENCES IN EFFICIENCY IN MULTIPLICATION RESULTING FROM DRILL ON THE NUMBER FACTS OF ADDITION

Subject	Difference in number attempted in Test 2 over Test 1		Difference in number correct in Test 2 over Test 1		Difference in number attempted in Test 3 over Test 1		Difference in number correct in Test 3 over Test 1		Difference in number attempted in Test 4 over Test 1		Difference in number correct in Test 4 over Test 1		Difference in number attempted in Test 3 over Test 2		Difference in number correct in Test 3 over Test 2		Difference in number attempted in Test 4 over Test 2		Difference in number correct in Test 4 over Test 2		Difference in number attempted in Test 4 over Test 3		Difference in number correct in Test 4 over Test 3	
	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent
1.....	6	26.57	7	43.75	23	109.52	15	93.75	10	47.62	12	75.00	17	62.96	18	78.26	4	14.81	5	21.74	—13	—29.53	—13	—31.71
2.....	6	10.35	—1	—1.62	6	10.35	7	12.73	6	10.35	7	12.73	0	00.00	8	14.81	0	00.00	8	14.81	0	00.00	0	00.00
3.....	12	23.08	17	37.77	12	23.08	16	35.56	12	23.08	16	35.56	0	00.00	—1	—1.61	0	00.00	—1	—1.61	0	00.00	0	00.00
4.....	0	00.00	9	16.98	0	00.00	9	16.98	0	00.00	9	16.98	0	00.00	—1	—1.61	0	00.00	—1	—1.61	0	00.00	0	00.00
5.....	16	57.14	12	50.00	12	42.75	10	41.67	14	50.00	15	62.50	—4	—9.09	—2	—5.56	—2	—4.55	3	8.33	2	5.00	5	14.71
6.....	12	30.00	13	38.24	16	40.00	15	44.12	13	32.50	16	47.06	4	7.69	2	4.26	1	1.92	—3	6.36	—5.36	1		
7.....	9	31.03	11	40.75	6	20.69	2	7.41	7	24.44	7	25.93	—3	—7.89	—9	—23.68	—2	—5.26	—4	—10.53	2.86	1	2.04	1
8.....	12	50.00	15	68.75	10	41.67	14	87.50	15	62.50	19	118.75	—2	—5.56	—1	—3.23	—3	8.33	—4	12.90	1	2.86	5	17.24
9.....	12	75.00	12	109.09	10	62.50	5	45.45	6	37.48	8	72.73	—7	—7.14	—7	—30.43	—6	—21.43	—4	—17.39	—4	—15.38	—5	—16.67
10.....	11	32.35	10	32.26	13	38.23	13	41.93	14	41.19	12	38.71	—2	—4.44	—3	—7.32	—3	—6.67	—2	—4.88	—1	—2.13	—1	—2.27
11.....	10	71.43	10	76.92	8	57.14	0	00.00	10	71.43	9	69.23	—2	—8.33	—10	—43.48	0	00.00	—1	—4.35	2	9.09	9	69.23
12.....	17	54.84	16	53.33	9	29.03	6	26.66	14	45.16	13	43.33	—8	—16.67	—6	—17.39	—3	—6.25	—3	—6.52	5	12.50	5	13.16
13.....	11	40.74	5	18.51	9	33.33	6	22.22	12	44.44	8	29.63	—3	—5.26	1	3.13	1	2.63	—3	9.38	3	8.33	3	6.06
14.....	7	36.84	9	52.94	8	42.10	9	52.94	6	31.56	7	41.18	1	3.85	0	00.00	—1	—3.85	—2	—7.69	—2	—7.41	—2	—7.69
15.....	16	45.71	16	55.56	11	31.43	14	46.27	9	22.86	8	27.58	—5	—9.80	—2	—4.44	—7	—13.73	—5	—17.78	—2	—4.35	—2	—6.95
16.....	13	56.52	15	107.14	10	43.52	13	92.85	17	73.91	25	178.51	—3	—8.33	—2	—6.90	4	11.11	10	34.48	7	21.21	12	44.44
17.....	21	62.50	21	67.74	19	47.50	16	51.61	16	40.00	19	61.29	—2	—3.28	—3	—9.62	—6	—8.20	—2	—3.85	—3	—5.08	3	6.38
18.....	4	11.11	4	11.11	7	19.44	6	16.77	7	19.44	7	19.44	3	7.50	2	5.00	3	7.50	3	7.50	0	00.00	1	2.38
19.....	6	6.00	3	6.27	9	18.00	9	18.75	10	20.00	7	14.58	6	11.76	6	11.32	7	13.21	4	7.84	1	1.99	—2	—3.51
20.....	18	43.90	17	41.28	13	31.71	15	38.46	15	36.59	13	33.33	—5	—8.47	—2	—3.57	—3	—5.08	—4	—7.14	2	3.70	—2	—3.70
21.....	4	14.29	4	15.38	3	10.71	3	11.54	4	14.29	5	19.23	—1	—3.13	—1	—3.33	0	00.00	1	3.33	1	3.23	2	6.90
22.....	13	50.00	13	54.17	8	30.77	7	29.17	14	53.85	15	62.50	—5	—12.82	—6	—16.22	1	2.56	2	5.41	6	17.65	8	25.81
23.....	14	70.00	15	83.33	18	65.00	12	66.67	20	100.00	22	122.22	—1	—2.94	—3	—9.09	6	17.65	7	21.21	7	21.21	10	33.33
Average	10.7	32.66	11.0	38.10	10.2	31.06	10.2	33.58	10.9	33.20	12.1	42.02	—0.5	—1.20	—0.8	—2.07	0.2	0.40	1.1	2.83	0.7	1.61	2.0	5.01

A comparison of the scores of Tests 2 and 3, Tests 2 and 4, and Tests 3 and 4 shows no significant transfer value of drill in the higher combinations to solving multiplication problems.

The average number of operations attempted and the average number of correct operations for each test are given in Table XV. The difference between the averages for Tests 1 and 2 of the number of operations attempted is statistically reliable, and the difference between the average number of correct operations is so nearly reliable that one may conclude that drill on the digits 0 to 9 produced a significant increase in efficiency in solving problems in multiplication. The reliabilities of the differences between the average scores on Tests 2 and 3 and Tests 3 and 4 are not high enough to be significant. The difference between the average scores of Tests 1 and 3 is less than that between Tests 1 and 2 and is less reliable. There is a slight increase in the reliability of the differences between the average number of correct operations for Tests 1 and 4, but there is a slight decrease in the reliability of the differences between the average number of columns attempted.

TABLE XV. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGES OF THE NUMBER OF OPERATIONS ATTEMPTED AND THE NUMBER OF OPERATIONS COMPLETED CORRECTLY IN MULTIPLICATION

	Test 1	Test 2	Test 3	Test 4	Test 1	Test 3	Test 2	Test 4
NUMBER ATTEMPTED—								
Average	32.9	43.6	43.1	43.8	32.9	43.1	43.6	43.8
S. D.	12.70	10.30	12.20	12.40	12.70	12.20	10.30	12.40
S. D. Average	2.67	2.17	2.54	2.58	2.67	2.54	2.17	2.58
Difference		10.7	0.5	0.7	10.9	10.2	0.5	0.2
S. D. Difference		3.44	3.34	3.62	3.71	3.69	3.34	3.37
Difference / S. D. Difference		3.11	0.15	0.19	2.93	2.73	0.15	0.06
NUMBER CORRECT—								
Average	28.9	39.9	39.0	41.0	28.9	39.0	39.9	41.0
S. D.	13.30	12.30	13.85	11.95	13.30	13.85	12.30	11.95
S. D. Average	2.74	2.56	2.88	2.49	2.74	2.88	2.56	2.49
Difference		11.0	0.9	2.0	12.1	10.1	0.9	1.1
S. D. Difference		3.75	3.85	3.81	3.70	3.97	3.85	3.57
Difference / S. D. Difference		2.93	0.23	0.52	3.29	2.52	0.23	0.31

The operations each subject solved correctly (Table XIII) on each of the four tests are plotted in the curves in Figure 10. An examination of the curves in Figure 10 shows clearly that drill on the addition combinations formed from the digits 0 to 9 produced a significant increase in efficiency in solving problems in multiplication. The proximity of the curves for Tests 2, 3, and 4 shows a lack of additional improvement resulting from drill in the higher number facts of addition.

Division.—The number of operations attempted and the number of operations correctly solved by the 23 subjects in the division problems for the four tests are given in Table XVI. There were three types of operations involved in the division problems: (1) division, (2) multiplication, and (3) subtraction. The total number of division, multiplication, and subtraction operations constitutes the score (the number of

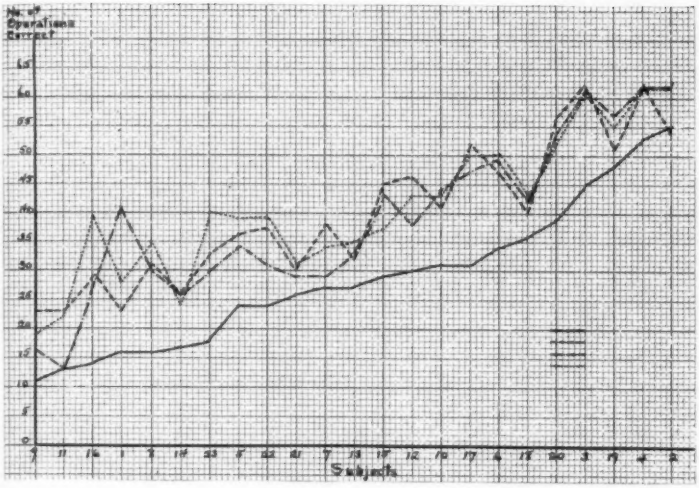

FIG. 10. Number of Multiplication Problem Operations Completed Correctly, Arranged According to the Scores Made on Test 1.

TABLE XVI. NUMBER OF OPERATIONS ATTEMPTED AND NUMBER OF OPERATIONS COMPLETED CORRECTLY IN DIVISION

Subject	Test 1		Test 2		Test 3		Test 4	
	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct	Number attempted	Number correct
1	24	22	31	30	23	21	27	27
2	68	66	75	73	70	69	72	71
3	56	55	65	63	70	70	56	54
4	68	66	75	75	72	68	72	68
5	36	35	49	45	44	42	54	50
6	43	42	55	51	46	45	54	54
7	42	39	48	45	64	59	56	55
8	46	44	48	47	42	39	54	53
9	36	36	31	30	48	47	36	35
10	36	36	42	42	54	53	44	43
11	5	4	22	20	34	31	30	29
12	42	42	54	53	68	66	72	71
13	30	30	39	39	34	33	46	46
14	32	31	31	30	45	43	46	46
15	52	52	75	72	68	66	69	68
16	33	32	37	34	49	46	44	41
17	64	62	75	72	72	70	72	72
18	48	47	43	43	63	63	60	58
19	54	54	60	60	66	66	72	72
20	62	61	75	73	72	72	72	71
21	40	38	40	40	43	42	48	46
22	56	56	50	49	56	56	52	48
23	22	22	25	24	28	27	30	30
Average	43.3	42.3	49.8	48.3	53.5	51.9	53.8	52.5

TABLE XVII. DIFFERENCES IN EFFICIENCY IN DIVISION RESULTING FROM DRILL ON THE NUMBER FACTS OF ADDITION

Subject	Difference in number attempted in Test 2 over Test 1		Difference in number correct in Test 2 over Test 1		Difference in number attempted in Test 3 over Test 1		Difference in number correct in Test 3 over Test 1		Difference in number attempted in Test 4 over Test 1		Difference in number correct in Test 4 over Test 1		Difference in number attempted in Test 3 over Test 2		Difference in number correct in Test 3 over Test 2		Difference in number attempted in Test 4 over Test 2		Difference in number correct in Test 4 over Test 2		Difference in number attempted in Test 4 over Test 3		Difference in number correct in Test 4 over Test 3	
	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent	Number	Per cent
1	7	29.17	8	36.36	—1	—4.17	—1	—4.55	3	12.50	5	22.75	—8	—25.81	—9	—30.00	—4	—12.90	—3	—10.00	4	17.39	6	28.57
2	7	10.17	7	10.61	2	2.92	3	4.55	4	5.88	5	7.58	—5	—6.67	—4	—5.48	—3	—4.00	—2	—2.74	2	2.86	2	2.89
3	9	16.07	8	14.55	14	25.00	15	27.27	0	00.00	—1	—1.82	5	7.69	7	11.11	—9	—13.84	—9	—14.28	—14	—20.00	—16	—22.85
4	7	10.29	9	13.64	4	5.88	2	3.03	4	5.88	2	3.03	—3	—4.00	—7	—9.33	—3	—4.00	—7	—9.33	0	00.00	0	00.00
5	13	36.11	10	28.57	8	22.22	7	20.00	18	50.00	15	42.86	—5	—10.20	—3	—6.67	5	10.20	5	11.11	10	22.73	8	19.05
6	12	27.90	9	21.43	3	6.98	3	7.14	11	25.58	12	28.57	—9	—16.36	—6	—11.76	—1	—1.82	3	5.88	8	17.39	9	20.00
7	6	14.29	6	15.38	22	52.38	20	51.28	14	33.33	16	41.03	16	33.33	14	31.11	8	16.67	10	22.22	—8	—12.50	—4	—6.78
8	2	45.65	3	6.82	—4	—8.70	—5	—11.36	8	17.39	9	20.45	—6	—12.50	—8	—17.02	6	12.50	6	12.77	12	28.57	14	35.79
9	—5	—13.89	—6	—16.67	12	33.33	11	30.56	0	00.00	—1	—2.78	17	54.84	17	56.67	5	16.13	5	16.67	—12	—25.00	—12	—28.57
10	6	16.67	6	16.67	18	50.00	17	47.22	8	22.22	7	19.44	12	28.57	11	26.19	2	4.74	1	2.37	—10	—18.52	—10	—18.87
11	17	340.00	16	400.00	29	580.00	27	675.00	25	500.00	25	625.00	12	54.55	11	55.00	8	36.36	9	45.00	—4	—11.76	—2	—6.45
12	12	28.57	11	26.19	26	61.90	24	57.14	30	71.43	29	69.05	14	25.93	13	24.53	18	33.33	18	33.96	4	5.88	5	7.57
13	9	30.00	9	30.00	4	13.33	3	10.00	16	53.33	16	53.33	—5	—12.82	—6	—15.38	7	17.95	7	17.95	12	35.29	13	39.39
14	—1	—3.13	—1	—3.23	13	40.63	12	38.71	14	43.75	15	48.39	14	45.16	13	43.33	15	48.39	16	53.33	1	2.22	3	6.98
15	23	44.23	20	38.46	14	26.92	14	26.92	17	32.69	16	30.77	—7	—9.33	—6	—8.33	—6	—8.00	—4	—5.56	1	1.47	2	3.03
16	4	12.12	2	6.25	16	48.48	14	43.75	11	33.33	9	28.13	12	32.43	12	35.29	7	18.92	7	20.51	—5	—10.20	—5	—10.87
17	11	17.19	10	16.13	8	12.50	8	12.90	8	12.50	10	16.13	—3	—4.00	—2	—2.78	—3	—4.00	0	00.00	0	00.00	2	2.86
18	—5	—10.42	—4	—8.51	15	31.25	16	34.04	12	25.00	11	23.40	20	46.51	20	46.51	17	39.51	15	34.89	—3	—4.76	—5	—7.93
19	6	11.11	6	11.11	12	22.22	12	22.22	18	33.33	18	33.33	6	10.00	6	10.00	12	20.00	12	20.00	6	9.09	6	9.09
20	13	20.97	12	19.67	10	16.13	11	18.03	10	16.13	10	16.39	—3	—4.00	—1	—1.37	—3	—4.00	—2	—2.74	0	00.00	—1	—1.39
21	0	00.00	2	5.26	3	7.50	4	10.53	8	20.00	8	21.05	3	7.50	2	5.00	8	20.00	6	15.00	5	11.63	4	9.52
22	—6	—10.71	—7	—12.32	0	00.00	0	00.00	—4	—7.14	—8	—14.29	6	12.00	7	14.29	2	4.00	—1	—2.04	—4	—7.14	—8	—14.28
23	3	13.64	2	9.09	6	27.27	5	22.73	8	36.36	8	36.36	3	12.00	3	12.50	5	20.00	6	25.00	2	7.14	3	11.11
Average	6.5	15.08	6.0	14.30	10.3	23.72	9.7	22.84	10.6	24.42	10.3	24.28	3.7	7.51	3.6	7.57	0.3	0.57	0.6	1.17	4.0	8.12	4.3	8.84

operations) for each test. The range in number of operations attempted for Test 1 is 5 to 68 and for correct operations 4 to 66. There is a considerable reduction in the range of scores on Test 2, the range for attempts being 22 to 75 and for corrects 20 to 75. There are no significant changes in the range of scores for Test 3, but on Test 4 there is a further reduction in the range of both attempts and corrects, since the low score made on this test was 27.

Individual changes in number of operations attempted and in number of correct operations in the division problems for Tests 2, 3, and 4 are given in Table XVII. Four subjects attempted fewer operations on Test 2 than on Test 1, but the group made an average gain of 15.08 per cent in number of operations attempted. There was a 14.30 per cent gain in number of correct operations. There was a further increase on Test 3 in the number of operations attempted and in correct operations of 23.72 and of 22.84 per cent respectively. There was only a slight additional increase in efficiency on Test 4 over Test 3. There was an average gain of 24.42 per cent in attempts and 24.28 per cent in correct solutions. On Test 3 there were 7.51 per cent more operations attempted and 7.57 per cent more correct operations than on Test 2. This increase is small when compared with the increase in scores on Test 2 over Test 1. The average increase on Test 4 over Test 2 is insignificant—0.57 per cent in attempts and 1.17 per cent in number of correct operations. The average increase in efficiency on Test 4 over Test 3 was 8.12 per cent in attempts and 8.84 per cent in corrects. This increase was not as large as that of Test 2 over Test 1, but was sufficiently large to be significant.

The average number of operations attempted and the average number of correct operations for each test are given in Table XVIII. An examination of the reliabilities of the differences between the averages for Test 1 and the subsequent three tests shows no statistically reliable difference. The reliability of the differences between the averages for Tests 1 and 3 and Tests 1 and 4 is high enough to be significant.

TABLE XVIII. RELIABILITY OF THE DIFFERENCES BETWEEN THE AVERAGES OF THE NUMBER OF OPERATIONS ATTEMPTED AND THE NUMBER OF OPERATIONS COMPLETED CORRECTLY IN DIVISION

	Test 1	Test 2	Test 3	Test 4	Test 1	Test 3	Test 2	Test 4
NUMBER ATTEMPTED—								
Average	43.3	49.8	53.5	53.8	43.3	53.5	49.8	53.8
S. D.	15.70	15.85	15.25	12.50	15.70	15.25	15.85	12.50
S. D. Average	3.27	3.30	3.18	2.60	3.27	3.18	3.30	2.60
Difference		6.5	3.7	0.3	10.5	10.2	3.7	4.0
S. D. Difference		4.65	4.59	4.11	4.18	4.56	4.59	4.20
Difference / S. D. Difference		1.39	0.81	0.07	2.41	2.23	0.81	0.95
NUMBER CORRECT—								
Average	42.3	48.3	51.9	52.5	42.3	51.9	48.3	52.5
S. D.	15.65	16.80	14.90	14.85	15.65	14.90	16.80	14.85
S. D. Average	3.27	3.50	3.10	3.10	3.27	3.10	3.50	3.10
Difference		6.0	3.6	0.6	10.2	9.6	3.6	4.2
S. D. Difference		4.78	4.68	4.38	4.50	4.50	4.68	4.68
Difference / S. D. Difference		1.25	0.77	0.14	2.27	2.13	0.77	0.88

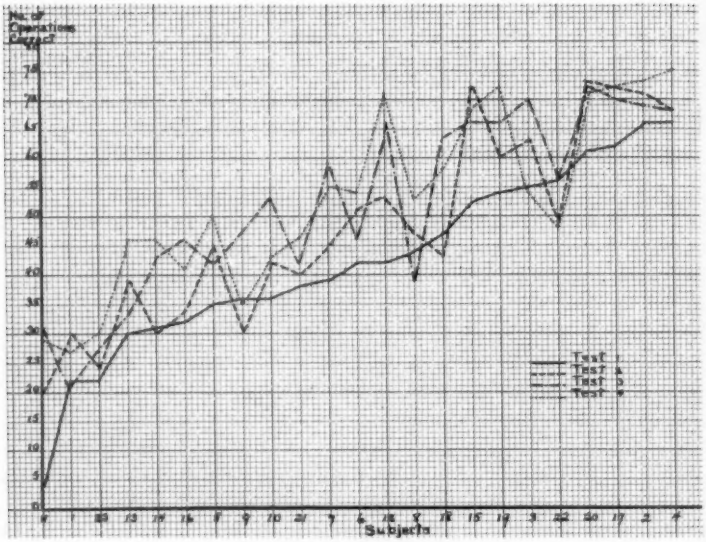

FIG. 11. Number of Division Problem Operations Completed Correctly, Arranged According to the Scores Made on Test 1.

The operations in the division problems that each subject solved correctly on the four tests are plotted in the curves in Figure 11. The greatest distance between curves is between the curves for Tests 1 and 2. This fact indicates that drill in the 0 to 9 combinations produced a significant increase in efficiency in solving problems in division. The proximity of the curves for Tests 2, 3, and 4 shows the lack of any great improvement resulting from the drill in the higher combinations.

A consideration of all the data presented concerning the performance of the subjects in the division problems indicates that the drill on the 0 to 9 combinations had a significant value and that the transfer effect of drill on the higher combinations was slight but positive.

COMPARISON OF CHANGES IN EFFICIENCY

Addition, subtraction, multiplication, and division.—A comparison of the changes in efficiency in problem computation in addition, subtraction, multiplication, and division resulting from drill in the number facts of addition presents some interesting data. The curves in Figure 12 represent the percentage gain in correct solutions for the four types of problems. Percentages are computed on the basis of the scores made on Test 1. The curve representing changes in adding efficiency is plotted from the average percentage changes for the 8-, 13-, and 18-digit problems for each test. An examination of the curves in Figure 12 shows that the transfer value of drill on the number facts of addition in solving

problems in subtraction, multiplication, and division was greater than the direct influence on computing problems in addition. To the subjects as a group the drill on the number facts of addition was of the greatest value in solving problems in multiplication, followed in order by division, subtraction, and addition.

The composite curve representing increased facility in addition shows a somewhat greater percentage increase on Test 3, following the drill on the combinations 0 to 49, than it shows on Test 2, following drill on the 0 to 9 combinations. There is approximately the same increase on Test 4 over Test 3 after drill on the combinations 0 to 99 as was registered on Test 2. The shape of the addition curve indicates that the teaching of the 1,000 number facts of addition is desirable since problem-solving ability increases with the number of combinations.

Drill on the combinations 0 to 9 produced significant increases in efficiency in solving problems in subtraction, multiplication, and division. Additional drill on the combinations 0 to 49 produced no significant changes in efficiency in solving problems in subtraction and multiplication but did facilitate the working of problems in division. Further drill on the combinations 0 to 99 increased efficiency in solving problems in subtraction and multiplication but had little effect on division. The shape of the curves for subtraction, multiplication, and division offers some evidence that there is a continued positive transfer value above that of the combinations 0 to 9 in teaching the higher combinations; that is, the more number facts of addition that are taught, the greater is the increase in efficiency in solving problems in subtraction, multiplication, and division.

Superior, average, and poor adders.—The questions have been raised as to whether superior, average, or poor adders receive the most benefit from drill on the higher number facts of addition and whether the transfer value to problem computation in subtraction, multiplication, and division varies with the degree of efficiency in addition. To simplify the

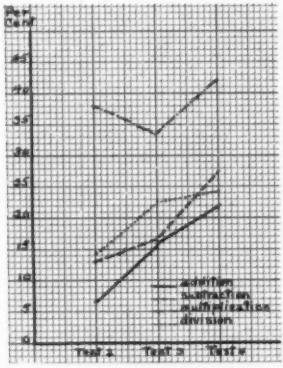

FIG. 12. Comparison of the Percentage Increase in Efficiency in Addition, Subtraction, Multiplication, and Division.

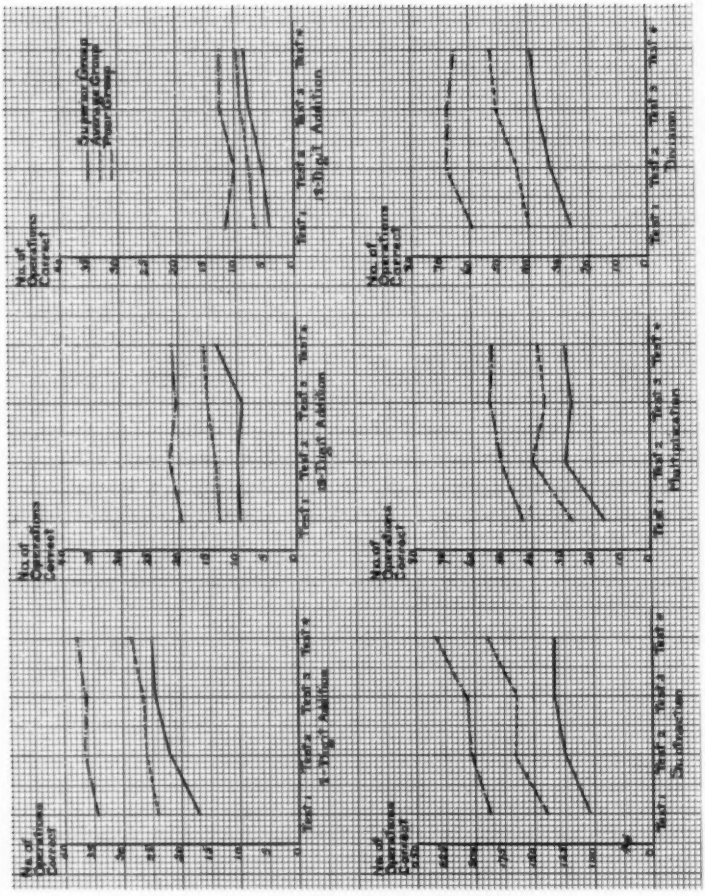

FIG. 13. Comparison of the Average Scores Made by the Superior, Average, and Poor Adders in Each Type of Problem.

study of these two problems, the scores (number of correct operations) on Test 1 in each type of problem were arranged in order from highest to lowest and divided into three groups: Group 1 (superior), the eight subjects having the highest scores; Group 2 (average), the seven subjects in the middle of the distribution; and Group 3 (poor), the eight subjects making the lowest scores.

The average scores made by the superior, average, and poor adders for each of the four tests for the 8-, 13-, and 18-digit problems and for the subtraction, multiplication, and division problems are given in Table XIX. A comparison of the data, however, may be made more readily by examining the curves in Figure 13. The curves for the three groups in the 8-, 13-, and 18-digit addition problems shows that the differences in efficiency existing in the initial performance (Test 1) remained relatively constant throughout the four tests. Variations between the three curves for the different tests are insignificant. The similarity of the shapes of the curves for the three groups in subtraction, multiplication, and division indicates that the transfer value from drill in the number facts of addition was approximately equal for superior, average, and poor adders.

TABLE XIX. AVERAGE SCORES MADE BY THE SUPERIOR, AVERAGE, AND POOR ADDERS IN EACH TYPE OF PROBLEM

	Test 1	Test 2	Test 3	Test 4
8-DIGIT ADDITION—				
Upper third	34.25	36.75	36.75	37.12
Middle third	23.85	26.00	26.71	28.43
Lower third	17.25	22.25	24.75	25.12
13-DIGIT ADDITION—				
Upper third	19.50	21.12	20.50	21.00
Middle third	13.00	13.57	14.00	14.14
Lower third	9.62	9.75	9.25	13.25
18-DIGIT ADDITION—				
Upper third	11.88	10.25	12.62	12.88
Middle third	6.86	7.71	9.00	9.14
Lower third	4.00	5.12	7.62	8.25
SUBTRACTION—				
Upper third	188.00	201.88	208.88	233.75
Middle third	141.86	167.14	167.86	190.57
Lower third	106.50	126.50	135.25	135.88
MULTIPLICATION—				
Upper third	42.62	51.62	53.88	53.50
Middle third	27.71	39.86	36.14	38.43
Lower third	16.12	30.62	26.75	30.75
DIVISION—				
Upper third	59.00	67.12	67.12	65.50
Middle third	41.14	44.14	51.57	53.14
Lower third	26.50	33.00	37.00	39.00

FIG. 13. Comparison of the Average Scores Made by the Superior, Average, and Poor Adders in Each Type of Problem.

SUMMARY AND CONCLUSIONS

In order to determine changes in efficiency in problem-solving ability in addition, subtraction, multiplication, and division resulting from drill on the number facts of addition, an unrelated group of 23 college students was given a series of four tests, each test consisting of three sets of two-column problems in addition (8, 13, and 18 digits high) and one set each of subtraction, multiplication, and division problems. Following Test 1, the subjects were drilled on the 100 combinations formed from the digits 0 to 9. Before taking Test 3 the subjects were drilled on the

combinations 0 to 49, and preceding the execution of the problems in Test 4 the subjects were drilled on the combinations 0 to 99. The important conclusions of this study are as follows:

1. Drill on the combinations 0 to 9 produced a significant increase in ability to add two-column problems 8 digits high. Further drill on the combinations 0 to 49 and 0 to 99 failed to produce significant increases in efficiency.

2. In the 13-digit problems the most significant increase in efficiency resulted from drill on the 0 to 99 combinations. Drill on the 0 to 9 combinations also increased adding ability but to a lesser extent than did the drill on the 0 to 99 combinations. Drill on the 0 to 49 combinations seemed not to be beneficial in adding two-column problems 13 digits high.

3. The only increase in adding ability which might be considered significant in the 18-digit problems was produced by drill on the 0 to 49 combinations. Drill on the combinations 0 to 9 and 0 to 99 did not produce significant increases in ability to add.

4. In considering the changes in efficiency for the three lengths of problems, there is some evidence, although not entirely conclusive, that adding efficiency will increase in proportion to the number of addition facts taught.

5. The most significant transfer value in subtraction from drill on the number facts of addition resulted from drill on the 0 to 9 and 0 to 99 combinations. The transfer value of drill on the combinations 0 to 49, although positive, is not great enough to be significant.

6. Drill on the combinations 0 to 9 produced a significant increase in ability to solve multiplication problems. Further drill on the higher combinations failed to produce significant increases in efficiency.

7. The most significant transfer value in solving problems in division from drill on the number facts of addition resulted from the 0 to 9 combinations. There was a small positive transfer from drill on the combinations 0 to 49 and 0 to 99, but not great enough to be highly significant.

8. The transfer value from drill on the number facts of addition was greater in solving problems in subtraction, multiplication, and division than it was in computing problems in addition.

9. The greatest transfer value was in solving problems in multiplication. Division and subtraction followed, in order.

10. There was considerable evidence that the teaching of more number facts of addition brought about greater increase in efficiency in solving problems in multiplication, subtraction, and division.

11. Changes in adding efficiency resulting from drill on the number facts of addition were approximately the same for superior, average, and poor adders.

12. The transfer value from drill on the number facts of addition to the solving of subtraction, multiplication, and division problems was approximately the same for the superior, average, and poor groups.

A List of Bulletins in the Field of Education, Indiana University

The following is a list of the bulletins published by the School of Education, Indiana University.

All bulletins which are available at the present time may be secured for fifty cents ($.50) per copy, with the exception of the *Second Revision of the Bibliography of Educational Measurements;* the *Bibliography of School Buildings, Grounds and Equipment—Part I; The Philosophy of Human Relations—Individual and Collective;* the *Bibliography of School Surveys and of References on School Surveys;* and the *Bibliography of College and University Buildings, Grounds, and Equipment,* which may be secured for seventy-five cents ($.75) per copy and may be obtained from the University Bookstore.

Proceedings of the High School Principals' Conference (November 23 and 24, 1923). Vol. I, No. 1, 1924. 85 p. (Supply exhausted.)

Investigation of Nursing as a Professional Opportunity for Girls. Part I, Technical Study; Part II, Vocational Information Bulletin. By Florence E. Blazier. Vol. I, No. 2, 1924. 69 p.

Proceedings of the Eleventh Conference on Educational Measurements. Vol. I, No. 3, 1925. 141 p.

Proceedings of the High School Principals' Conference (November 14 and 15, 1924). Vol. I, No. 4, 1925. 49 p. (Supply exhausted.)

First Revision of the Bibliography of Educational Measurements. Compiled by the Bureau of Coöperative Research. Vol. I, No. 5, 1925. 147 p. (Supply exhausted.)

Proceedings of the Twelfth Conference on Educational Measurements. Vol. I, No. 6, 1925. 76 p.

The Effect of Population upon Ability to Support Education. By Harold F. Clark. Vol. II, No. 1, 1925. 28 p.

Proceedings of the High School Principals' Conference (November 20 and 21, 1925). Vol. II, No. 2, 1925. 77 p. (Supply exhausted.)

A Cross-Indexed Bibliography on School Budgets. By Harold F. Clark. Vol. II, No. 3, 1926. 66 p.

A Comparison of the Results Made on Certain Standardized Tests by Pupils in the Bloomington High School Who Were Taught in Classes of the Same Grade by University Student Teachers and by Regular High School Teachers. By Carl G. F. Franzén. Vol. II, No. 4, 1926. 19 p.

Proceedings of the Thirteenth Annual Conference on Educational Measurements. Vol. II, No. 5, 1926. 103 p.

When to Issue School Bonds. By Harold Florian Clark and Paul Royalty. Vol. II, No. 6, 1926. 16 p.

Students' Attitude Toward Examinations. By Grover T. Somers. Vol. III, No. 1, 1926. 48 p.

Proceedings of the High School Principals' Conference (November 12 and 13, 1926). Vol. III, No. 2, 1926. 27 p.

Index Numbers in School Administration. By Harold F. Clark. Vol. III, No. 3, 1927. 35 p.

Topical Analysis of 234 School Surveys. Compiled by the Bureau of Coöperative Research. Vol. III, No. 4, 1927. 111 p. (Supply exhausted.)

Proceedings of the Fourth Annual Conference on Elementary Supervision. Vol. III, No. 5, 1927. 64 p.

Proceedings of the Fourteenth Annual Conference on Educational Measurements. Vol. III, No. 6, 1927. 66 p.

Some Phases of the Junior College Movement. By I. Owen Foster, Harold F. Clark, Willard W. Patty, and Leo M. Chamberlain. Vol. IV, No. 1, 1927. 125 p. (Supply exhausted.)

Second Revision of the Bibliography of Educational Measurements. By Henry Lester Smith and Wendell William Wright. Vol. IV, No. 2, 1927. 251 p.

Bibliography of School Buildings, Grounds, and Equipment, Part I. By Henry Lester Smith and Leo Martin Chamberlain. Vol. IV, No. 3, 1928. 326 p.

Proceedings of the High School Principals' Conference (November 18 and 19, 1927). Vol. IV, No. 4, 1928. 54 p.

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Proceedings of the Fifteenth Annual Conference on Educational Measurements. Vol. IV, No. 6, 1928. 73 p.

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The Development and Use of a Composite Achievement Test. By Wendell William Wright. Vol. V, No. 3, 1929. 90 p.

An Analysis of the Attitudes of American Educators and Others Toward a Program of Education for World Friendship and Understanding. By Henry Lester Smith and Leo Martin Chamberlain. Vol. V, No. 4, 1929. 109 p.

Tentative Program for Teaching World Friendship and Understanding in Teacher Training Institutions and in Public Schools for Children Who Range from Six to Fourteen Years of Age. By Henry Lester Smith and Sherman Gideon Crayton. Vol. V, No. 5, 1929. 54 p.

Proceedings of the Sixteenth Annual Conference on Educational Measurements. Vol. V, No. 6, 1929. 96 p.

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An Analysis of the Duties of County School Superintendents and Superintendents of Schools in Certain Cities in Indiana. By Henry Lester Smith and Leo Martin Chamberlain. Vol. VI, No. 2, 1929. 94 p.

Proceedings of the High School Principals' Conference (November 22 and 23, 1929). Vol. VI, No. 3, 1930. 51 p.

Coöperative Studies in Secondary Education. By Henry Lester Smith and Carl G. F. Franzén. Vol. VI, No. 4, 1930. 121 p.

Proceedings of the Seventeenth Annual Conference on Educational Measurements. Vol. VI, No. 5, 1930. 103 p.

Proceedings of the Seventh Annual Conference on Elementary Supervision. Vol. VI, No. 6, 1930. 102 p.

A Study in Teacher Supply and Demand in Indiana. By I. Owen Foster, Robert K. Devricks, Harry N. Fitch, Earl C. Bowman, and George L. Roberts. Vol. VII, No. 1, 1930. 77 p.

Proceedings of the High School Principals' Conference (November 7 and 8, 1930). Vol. VII, No. 2, 1930. 70 p.

The Philosophy of Human Relations: Individual and Collective—A Source Book. By Henry Lester Smith and Harold Littell. Vol. VII, No. 3, 1931. 326 p.

The Psychology of Human Relationships: Individual and Social—A Source Book. By Henry Lester Smith and Levi McKinley Krueger. Vol. VII, No. 4, 1931. 107 p.

Proceedings of the Eighteenth Annual Conference on Educational Measurements. Vol. VII, No. 5, 1931. 83 p.

Efficiency in College Management. By John Dale Russell. Vol. VII, No. 6, 1931. 143 p.

Bibliography of School Surveys and of References on School Surveys. By Henry Lester Smith and Edgar Alvin O'Dell. Vol. VIII, Nos. 1 and 2, 1931. 212 p.

Proceedings of the Nineteenth Annual Conference on Educational Measurements. Vol. IX, No. 1, 1932. 139 p.

Bibliography of School Buildings, Grounds, and Equipment—Part II. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. IX, No. 2, 1933. 182 p.

Bibliography of School Buildings, Grounds, and Equipment—Part III. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. IX, No. 3, 1933. 130 p.

A Brief Summary of Literature on Leadership. By Henry Lester Smith and Levi McKinley Krueger. Vol. IX, No. 4, 1933. 80 p.

Proceedings of the Twentieth Annual Conference on Educational Measurements. Vol. X, No. 1, 1933. 98 p.

Bibliography of College and University Buildings, Grounds, and Equipment. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. X, No. 2, 1934. 199 p.

Factors Affecting the Organization of School Attendance Units. By Raleigh Warren Holmstedt. Vol. X, No. 3, 1934. 32 p.

The Old World Historical Background of Religious and Moral Education in Schools. By Henry Lester Smith, Robert Stewart McElhinney, and George Renwick Steele. Vol. X, No. 4, 1934. 144 p.

Proceedings of the Twenty-first Annual Conference on Educational Measurements. Vol. XI, No. 1, 1934. 96 p.

Bibliography of School Buildings, Grounds, and Equipment—Part IV. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. XI, No. 2, 1935. 216 p.

A Brief Survey of Present-Day Religious and Moral Education in the Schools of Countries Other Than the United States of America. By Henry Lester Smith, Robert Stewart McElhinney, and George Renwick Steele. Vol. XI, No. 3, 1935. 185 p.

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Proceedings of the Twenty-second Annual Conference on Educational Measurements. Vol. XII, No. 1, 1935. 97 p.

A Basis for the Improvement of Education in Rural Monroe County, Indiana. By Henry Lester Smith and Forest Ruby Noffsinger. Vol. XII, No. 2, 1936. 168 p.

Reading Readiness—A Prognostic Study. By Wendell William Wright. Vol. XII, No. 3, 1936. 46 p.

Proceedings of the Twenty-third Annual Conference on Educational Measurements. Vol. XII, No. 4, 1936. 60 p.

Bibliography of Literature on the Teaching of English. By Henry Lester Smith and William Isaac Painter. Vol. XIII, No. 1, 1936. 299 p.

Bibliography of Literature on Education in Countries Other Than the United States of America. By Henry Lester Smith and William Isaac Painter. Vol. XIII, No. 2, 1937. 341 p.

Character Development through Religious and Moral Education in the Public Schools of the United States. By Henry Lester Smith, Robert Stewart McElhinney, and George Renwick Steele. Vol. XIII, No. 3, 1937. 137 p.

Proceedings of the Twenty-fourth Annual Conference on Educational Measurements. Vol. XIII, No. 4, 1937. 100 p.

Bibliography of Literature on Education in Countries Other Than the United States of America (January 1, 1919, to December 31, 1924). By Henry Lester Smith and William Isaac Painter. Vol. XIV, No. 1, 1937. 139 p.

Two Studies in Education. By Henry Lester Smith and Merrill Thomas Eaton. Vol. XIV, No. 2, 1938. 42 p.

Bibliography of School Surveys and of References on School Surveys (Supplement to Vol. VIII, Nos. 1 and 2). By Henry Lester Smith and Edgar Alvin O'Dell. Vol. XIV, No. 3, 1938. 144 p.

Proceedings of the Twenty-fifth Annual Conference on Educational Measurements. Vol. XIV, No. 4, 1938. 37 p.

Remedial Instruction in Reading With College Freshmen. By Nila Banton Smith. Vol. XV, No. 1, 1938. 35 p.